Temporary Contracts as a Screening Device in the Frictional Labour Market*

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Abstract: In this paper, we develop a matching model with both permanent and temporary contracts to address situations in which the quality of a match formed by a worker–firm pair is not observable to both workers and firms. The screening and cost-saving aspects of temporary employment contracts are two primary reasons that firms use them, but screening has received little attention in the study of employment protection. We show that a reduction in dismissal costs increases job creation and decreases the difference between hiring thresholds of permanent and temporary employees. The latter result implies that temporary contracts are more likely to be used as a screening device in countries with stringent employment protections. We also examine how a change in the probability that the true quality of a match is revealed affects firms’ hiring decisions with respect to both types of contracts.

Key words: temporary contracts, screening, dismissal costs, hiring policies.

JEL: Classification: J41, J63, J64

1. Introduction

One of the most important recent topics in labour economics is the issue of how employment protection legislation affects labour markets. In Europe, high and persistent unemployment rates (compared to those in the U.S.) are thought to result from stringent employment protection that has generated labour market rigidities. In the 1980s, many European countries addressed this problem by liberalising the use of temporary contracts, with the aim of combating unemployment. However, introducing flexible employment contracts into economies with high unemployment produced only inconclusive results and remains theoretically and empirically controversial. Theoretical models predict that more stringent employment protection reduces both job creation and job destruction, which makes the overall effect on employment (and unemployment) ambiguous. This effect may imply that more flexible regulation of temporary employment may create new jobs but that these jobs

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are not well protected by employment legislation and are therefore unstable. In certain cases, the latter effect dominates the former, and the unemployment rate rises (see Blanchard and Landier (2002) and Cahuc and Postel-Vinay (2002)).

There are several reasons why employers use fixed-term employment contracts. Portugal and Varejao (2009) identify three reasons for the use of such contracts: (i) saving on future termination costs (and certain fringe benefits); (ii) temporary replacement; and (iii) screening for permanent positions. The first two reasons relate directly to firing costs. If firms are required to pay lower (or no) firing costs with a fixed-term contract when employees are dismissed, hiring a temporary worker will save money. Furthermore, firms might replace incumbent workers with temporary workers and assess the adequacy of their employees for permanent positions. In this regard, temporary replacement enables screening of employees. If fixed-term contracts are used to screen employees, then firms can learn about the employee and decide whether the current match should be converted into a permanent form. While it is controversial whether temporary contracts are stepping-stones to permanent contracts or dead-ends, a considerable number of temporary workers are currently being hired into longer duration contracts in certain European countries (see OECD (2002)). The evidence that temporary work can be a stepping-stone to permanent work suggests that a screening effect is being exploited by firms with respect to such temporary workers. Although several studies have incorporated the distinction between contract types into theoretical models that examine the effects of employment protection legislation on labour markets, the screening role of temporary contracts has not been a focus of research attention. Boockmann and Hagen (2008) and Portugal and Varejao (2009) find indications that temporary contracts help screen workers for permanent positions. Even if a match is revealed to be unproductive, firing a worker from a permanent job is costly in countries with stringent employment protections. The screening role of temporary contracts is thus indispensable in such countries. Thus, the purpose of this paper is to theoretically examine how employment protection for permanent jobs affects the hiring decisions of firms when the quality of an employment match (the productivity of a worker–firm pair) is match-specific.

1) Booth et al. (2002) present evidence that temporary work is a stepping-stone to permanent work, depending on the type of contract and gender of the employee. Ichino et al. (2008) find that a job at a temporary work agency may be an effective springboard to a permanent contract in Europe although not in the U.S.
2) Several studies focus on this topic: Sara (2007), Albertini et al. (2009), Bucher (2010) and Faccini (2013).
3) Beckmann and Kuhn (2012) address the issue of whether there are performance differences between establishments that use temporary agency workers as a buffer stock (called the flexibility strategy) and establishments that use them for screening purposes. They conclude that the productivity of establishments that use the flexibility strategy is significantly lower than that of establishments that use the screening strategy.
and not perfectly observable.

The equilibrium search model is helpful in studying the effects of employment protection when both permanent and temporary jobs are considered. In particular, the endogenous job destruction framework constructed by Mortensen and Pissarides (1994) is a standard model for study in this field. In the basic model with endogenous job destruction, the productivity of each job is characterised as a random shock, and the decisions of a firm and a worker depend on the value of productivity. Thus, once a productivity shock occurs, both the firm and the worker can observe it. Under this setup, conversion of a temporary contract to a permanent contract also depends on the realised value of the productivity shock. With respect to the screening role of temporary contracts, such a framework is unsuitable because the productivity of a match is known after the shock, and there is no need to screen workers. In other words, screening is significant for firms when information about a worker type or match quality is not fully revealed.4)

The present paper has a motivation similar to that of Faccini (2013), which extends the model of Pries and Rogerson (2005) by introducing permanent and temporary forms of employment contracts and showing that this type of model can account for the high transition rates from temporary to permanent employment in some European countries.5) Pries and Rogerson (2005) originally considered a situation in which only a publicly observable signal regarding the quality of a worker–firm match is obtained at the time of meeting, while the true quality of the match is revealed over time after the match has been formed and work has begun.6) Faccini (2013) adopts this learning mechanism to develop a model in which temporary contracts are used as screening devices.

Although the motivation for this paper is similar to that of Faccini (2013), there are two

4) Based on the notion that some aspects of a worker–firm match can only be revealed after the employment relationship has been established, Nagypal (2002) focus on the difference in the learning process between learning about match quality and learning by doing and examine how an economy's average productivity is affected by the imposition of dismissal costs. Using a model of heterogeneous workers that incorporates search frictions and endogenous separations of employment matches, Ravenna and Walsh (2012) examine how the heterogeneity of workers amplifies unemployment fluctuations, depending on the size of gross labour flows and asymmetric productivity shocks. In their model, a worker's type is revealed by interviewing and screening in the job search and recruitment process.

5) Several studies extend the framework of Pries and Rogerson (2005). Sara (2007) incorporates job-specific shocks and examines the impact of employment protection on labour productivity. In Sara (2007), the productivity of a job match is composed of a job-specific component and a match-specific component, where the former is observable, but the latter is unobservable. Regarding the match-specific component, her model is based on the same process of learning about the true quality of the match. Albertini et al. (2009) focus on the screening effect of temporary jobs on transitions to regular employment. In their model, the effects of hiring and firing subsidies on the unemployment rate and social welfare are examined. Bucher (2010) investigates the conditions under which temporary jobs are stepping stones to permanent jobs.
major differences in the structure of the models. First, Faccini (2013) assumes that firms can offer temporary employment with exogenous probability, which represents a restriction on the use of temporary contracts. In this paper, however, we assume that firms endogenously choose what type of contract to offer. This assumption enables us to examine how a change in regulations regarding the termination of permanent employment contracts distorts firms' optimal hiring decisions. Second, in Faccini (2013), a firm is allowed to maintain an employee in a temporary position continuously if the match quality is not revealed and a negative economic shock does not occur. However, we do not permit this possibility, and all temporary jobs can persist for only one period. At the end of each period, firms with an employee in a temporary position must decide whether they wish to convert the current employment contract into a permanent contract or terminate the employment relationship.

In addition, we note that the endogenous choice between permanent and temporary jobs is a key element of this model. As noted by Cahuc et al. (2012), the prior literature assumes that temporary jobs are preferable to permanent ones and that all new jobs start as temporary jobs or that regulation forces firms to create permanent jobs. However, various regulations on the use of temporary contracts—pertaining, for example, to valid reasons for the use of FTCs (fixed-term contracts), the maximum number of successive FTCs and the maximum cumulative duration of successive FTCs—are imposed in many OECD countries (see the employment protection legislation database of ILO). As a result of these regulations, offering temporary contracts may not always be the best choice. Accordingly, we assume that a temporary job lasts only one period\(^7\) and show that the types of contracts chosen depend on the realised value of the observed signal of a match type. As research regarding the choice between temporary and permanent jobs (combined with a screening role of temporary jobs) is limited, our paper offers new insights into the impact of employment protection.\(^8\)

Three main results are obtained. First, a reduction in dismissal costs with respect to permanent jobs increases job creation. Lower dismissal costs reduce the hiring threshold for

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6) Kugler and Saint-Paul (2004) develop a matching model in which adverse selection and firing costs are incorporated, examining reemployment probabilities of employed and unemployed workers. Because employers can only observe worker quality imperfectly, they may hire workers with poor match quality. Kugler and Saint-Paul show that the introduction of unjust dismissal provisions reduces the reemployment probabilities of unemployed job seekers but not that of employed job seekers.

7) Cahuc et al. (2012) note that in France, the average duration of temporary jobs is quite short (about one and a half months) and construct a job search and matching model with different expected durations to capture the choice of employers regarding the forms of contracts.

8) Kahn (2010) uses the two-period model to investigate the impact of easing the creation of temporary jobs and shows that temporary contracts are more advantageous for employers during recessions than booms. Cao et al. (2011) extend Mortensen and Pissarides (1994), incorporating on-the-job search and permanent and temporary contracts into their model. They show that an increase in firing costs increases wage inequality and decreases the unemployment rate.
each type of employment contract, and firms with vacancies can thus form matches in shorter periods. This increases the expected profit of vacancies and job creation. Second, a reduction in dismissal costs decreases the difference between hiring thresholds of permanent and temporary contracts. Although the effect of changes in dismissal costs on the hiring thresholds of each employment contract is indeterminate, this result implies that employers will be less selective in hiring a worker through permanent contracts. In other words, temporary contracts are more likely to be used as a screening device in countries with stringent employment protections. A numerical exercise shows that higher dismissal costs reduce the hiring threshold for temporary jobs and raise the threshold for permanent jobs. Third, we examine how a change in the probability that the true quality of a match is revealed affects firms’ hiring decisions with respect to both temporary and permanent jobs. According to the numerical exercise, a rise in this probability lowers the hiring threshold for temporary contracts. In contrast, the effect of a rise in the probability of revelation on the threshold for permanent jobs can be either positive or negative, and the timing of a change in its sign depends on dismissal costs (the effect is negative for low values of the revelation probability but turns positive for higher values).

The remainder of the paper is organised as follows. In Section 2, the basic framework of the model is described. In Section 3, a steady–state equilibrium is characterised. In Section 4, we investigate how dismissal costs affect job creation and the hiring thresholds for each type of contract. Finally, Section 5 concludes.

2. Model

2.1 Description of the economy

Following Faccini (2013), we extend the model studied by Pries and Rogerson (2005) to allow for two types of employment contracts: temporary contracts and permanent contracts. We employ a discrete–time framework and assume that both workers and firms discount the future by a constant rate β (which is a discount factor). There are many workers who are either employed or unemployed, and the measure of workers is normalised to one. On–the–job search is ruled out, and thus only unemployed workers search for jobs. Firms are measured by the free entry/exit condition. All workers and firms are assumed to be risk–neutral.

This model explicitly includes labour market friction; therefore, job seeking and recruiting activities are time consuming. We assume that the meeting process is described by a constant–returns–to–scale matching technology, \( m(u, v) \), where \( u \) and \( v \) are measures of unemployed workers and vacancies, respectively. During each period, a firm with a vacancy
encounters a job seeker with probability \( m(u, v) / v \), and an unemployed worker encounters a vacancy with probability \( m(u, v) / u \). Let the former probability be denoted by \( q(\theta) \) and the latter probability denoted by \( \theta q(\theta) \), where \( \theta = v / u \) denotes labour market tightness. By assuming the constant-returns-to-scale matching technology, these probabilities can be represented as functions only of market tightness, \( \theta \). We assume that \( q(\theta) \to 1 \) and \( \theta q(\theta) \to 0 \) as \( \theta \to 0 \), and \( q(\theta) \to 0 \) and that \( \theta q(\theta) \to 1 \) as \( \theta \to \infty \).

The production technology and the learning processes regarding match quality are based on Pries and Rogerson (2005) and Faccini (2013). We first consider the production technology; a unit of production is a matched worker–firm pair, and the productivity of each job is match specific.\(^9\) As in the above literature, match-specific productivity is observed at the end of the period; it is represented by \( y = \tilde{y} + \varepsilon \), where \( \varepsilon \) is a zero-mean independently and identically distributed random variable and follows a uniform distribution with support \([ -z, z]\). \( \tilde{y} \) is the true quality of a match, which is either high or low; matches with high productivity are represented by \( \tilde{y} = y_h \), and matches with low productivity are represented by \( \tilde{y} = y_l \). Owing to the presence of the noise term \( \varepsilon \), neither the worker nor the firm can observe the true productivity of the match.

Match quality is considered to be both an inspection and an experience good. When a job seeker and a vacant firm meet, they observe a signal \( \pi \), which represents the probability that the match will be good. This signal is drawn from the cumulative distribution \( G(\pi) \) and is independent across matches. We assume that firms make hiring decisions based only on the realisation of \( \pi \). After forming a match, a worker and a firm observe productivity \( y \) and update their information about the true match quality. If realised productivity falls in the range \( [y_l + z, y_h + z] \), the worker and firm learn that the match has high productivity. Similarly, if realised productivity falls in the range \( [y_l - z, y_h - z] \), the match is revealed to be of low quality. If realised productivity is in the range \( [y_h - z, y_l + z] \), nothing is learned (we assume that \( y_h - z < y_l + z \)). Thus, the learning process is a form of “all-or-nothing”. Let \( a = (y_h - y_l) / 2z \) denote the probability that the true match quality is revealed. In this setting, a match of an unknown type with prior probability \( \pi \) will be revealed as high productivity with probability \( a \pi \) and will be revealed as low productivity with probability \( a(1 - \pi) \). We suppose that the quality of a match is unchanged when the match is retained.

### 2.2 Contracts

In this model, two types of contracts are considered: temporary contracts and perma-
tent contracts. In addition, there are two states of permanent contracts: preexisting and newly created. When preexisting permanent workers are dismissed, firms must pay fixed dismissal costs (firing taxes) \( d > 0 \). However, no cost is imposed when new permanent workers and temporary workers are dismissed. The transfer component of dismissal costs is not considered in this paper.\(^{10}\)

Separations from permanent jobs occur (i) when the match quality is revealed to be bad or (ii) when the economy turns out to be poor with constant probability \( \lambda \). With respect to temporary jobs, however, we assume that employees are dismissed if the true quality of the match is not revealed. In this regard, we implicitly assume that the maximum length of temporary contracts is legally limited and that a firm with a temporary job must decide whether to rehire its employee through a permanent contract or dismiss him.\(^{11}\)

### 2.3 Bellman equations

To derive the values of firms with each type of contract, some notation must be defined. Let us denote the value of a firm having a temporary job and a signal \( \pi \) by \( J_T(\pi) \) and the value of a firm having a permanent job and a signal \( \pi \) by \( J_I(\pi) \), where \( I \) is an indicator function that takes zero for a new permanent match and one for a preexisting match. The value of a vacant job is denoted by \( V \).

The expected value of a firm having a temporary job and a signal \( \pi \) is represented by

\[
J_T(\pi) = \pi y_h + (1 - \pi)y_l - w_T(\pi) + \beta(\lambda V + (1 - \lambda)[a(\pi J_d(1) + (1 - \pi) V) + (1 - a) V]),
\]

where \( w_T \) is the wage paid for a temporary job. As defined above, \( \pi \) is the probability that a match will be good, and flow productivity is given by the expected value, using \( \pi \). At the end of the period, a match-specific shock is realised, with the employer learning that the true quality of a match is high with probability \( a \pi \) and low with probability \( a(1 - \pi) \). With probability \( 1 - a \), the quality of a match is not revealed. In this case, the prediction regarding the quality of a match cannot be revised, and the employment relationship is terminated.

\(^{10}\) The relative size and importance of each component (the firing tax component and the transfer component) differ across countries. Garibaldi and Violante (2005) posit that the transfer component is not negligible in Italy, whereas the tax component of dismissal costs is considered to be substantially larger than the transfer component. They argue that the neutrality of severance payments, which was shown by Lazear (1990), continues to hold in their framework.

\(^{11}\) We will provide more details regarding job conversion below. This model is based on the idea that firms screen eligible workers for permanent positions. Faccini (2013) assumes that a match with an unknown type in a temporary job can be retained in the next period with exogenous probability. In our model, however, we assume that job conversion from temporary to permanent does not occur when the true match quality is unknown.
because temporary contracts cannot be renewed in the next period.

The expected value of a firm with a permanent job and signal \( \pi \) is represented by
\[
J_f(\pi) = \pi y_k + (1 - \pi) y_l - w_l(\pi) + \beta(\lambda V - d)
+ (1 - \lambda)[a(\pi J_f(1) + (1 - \pi)(V - d)) + (1 - a) J_f(\pi)],
\]
where \( w_l \) is the wage paid for a permanent job of type \( I(I=0, 1) \). Newly created matches are not covered by employment protection, and the outside option of those matches therefore does not include dismissal costs. Conversely, all matches are covered by employment protection one period later, and employers must pay dismissal costs for firing workers.

The expected value of a vacant firm is represented by
\[
V = -c + \beta \left\{ q(\theta) \left[ \int_0^{\pi_T} V dG(\pi) + \int_{\pi_T}^{\pi_T} J_f(\pi) dG(\pi) + \int_{\pi_T}^{\pi_T} J_g(\pi) dG(\pi) \right] + (1 - q(\theta)) V \right\},
\]
where \( c \) denotes the flow recruiting costs, and \( \pi_j(j=P, T) \) is the hiring threshold for contract type \( j \) (\( P \) represents “permanent,” and \( T \) represents “temporary”). We here assume that \( \pi_T > \pi_T \), as formally shown below. In equation (3), the employers’ choice regarding whether an employment match is designated as permanent or temporary depends on the realisation of the signal \( \pi \).

Let us denote the value of being employed in a temporary job with signal \( \pi \) as \( W_T(\pi) \). The value of being employed in a permanent job with signal \( \pi \) and the value of being unemployed are denoted by \( W_I(\pi) \) and \( U \), respectively; \( I \) is the indicator function that was defined above. The expected value of being employed in a temporary job with signal \( \pi \) is represented by
\[
W_T(\pi) = w_T(\pi) - \gamma + \beta(\lambda U + (1 - \lambda)[a(\pi W_T(1) + (1 - \pi) U) + (1 - a) U]),
\]
where \( \gamma \) represents the constant disutility of work.

The expected value of being employed in a permanent job with signal \( \pi \) is represented by
\[
W_T(\pi) = w_T(\pi) - \gamma + \beta(\lambda U + (1 - \lambda)[a(\pi W_T(1) + (1 - \pi) U) + (1 - a) W_T(\pi)]),
\]
and the expected value of being unemployed is represented as follows:
\[
U = \beta \left\{ q(\theta) \left[ \int_0^{\pi_T} U dG(\pi) + \int_{\pi_T}^{\pi_T} W_T(\pi) dG(\pi) + \int_{\pi_T}^{\pi_T} W_0(\pi) dG(\pi) \right] + (1 - q(\theta)) U \right\}.
\]

### 2.4 Surplus of a match

Let us define the joint surplus generated from forming a match as follows:
\[
S_I(\pi) = J_f(\pi) + W_I(\pi) - (V - I d) - U, \text{ for } I = 0, 1,
S_T(\pi) = J_f(\pi) + W_T(\pi) - V - U,
\]
where each equation is evaluated at any \( \pi \in [0, 1] \). The first equation represents the surplus
of a permanent job with current status $I$. The second equation represents the surplus of a temporary job. The type of match formed depends on the level of the observed signal $\pi$. We suppose that the wage in each job is determined by a standard asymmetric Nash bargaining process and denote the worker’s bargaining power by $\eta \in (0, 1)$. Under this wage determination mechanism, the worker and the firm that form a match divide the surplus according to the following sharing rule:

$$J_I(\pi) - (V - Id) = (1 - \eta) S_I(\pi) \text{ and } W_I(\pi) - U = \eta S_I(\pi), \text{ for } I = 0, 1,$$

$$J_T(\pi) - V = (1 - \eta) S_T(\pi) \text{ and } W_T(\pi) - U = \eta S_T(\pi).$$

Using these sharing rules, we can derive the expressions for joint surplus as follows. First, the values of joint surplus for permanent jobs are given by

$$S_0(\pi) = \pi y_h + (1 - \pi) y_l - \gamma + \beta(1 - \lambda) [a \pi S_0(1) + (1 - a) S_0(\pi)] - \frac{\eta c \theta}{\beta(1 - \eta)} - \beta d,$$

$$S_1(\pi) = \pi y_h + (1 - \pi) y_l - \gamma + \beta(1 - \lambda) [a \pi S_1(1) + (1 - a) S_1(\pi)] - \frac{\eta c \theta}{\beta(1 - \eta)} + (1 - \beta) d,$$

where we have used the free-entry condition that the values of vacancies are equal to zero: $V = 0$. Second, the surplus $S_T(\pi)$ generated by forming a match in the temporary form is represented by

$$S_T(\pi) = \pi y_h + (1 - \pi) y_l - \gamma + \beta(1 - \lambda) a \pi S_0(1) - \frac{\eta c \theta}{\beta(1 - \eta)}.$$

3. Equilibrium

3.1 Hiring decision

We first consider the hiring decisions of firms with permanent jobs. A firm decides to hire a worker on a permanent basis if $\pi \geq \pi^*_p$. The corresponding condition for optimal hiring is given by $S_0(\pi^*_p) = S_T(\pi^*_p)$. To obtain a concrete expression for this condition, it is useful to express $S_I(\pi)$ in a more tractable form. From (10), we obtain

$$S_I(\pi) = \pi y_h + (1 - \pi) y_l - \gamma + (1 - \beta) d - \frac{\eta c \theta \beta(1 - \eta)}{1 - \beta(1 - \lambda)(1 - a)} + \beta(1 - \lambda) a \pi S_0(1).$$

Utilising (9), (10) and (12), the following two results are derived:

$$S_0(\pi) = S_I(\pi) - d,$$

$$S_1(\pi) = \frac{y_h - y_l + \beta(1 - \lambda) a S_0(1)}{1 - \beta(1 - \lambda)(1 - a)}.$$

The first result depicts the relationship between $S_0(\pi)$ and $S_I(\pi)$. The surplus generated by a preexisting permanent job is equal to the sum of the surplus generated by a newly created permanent job and the cost of dismissal. The second result indicates that $S_I(\pi)$ is indepen-
dent of $\pi$, where $S'_i(\pi)$ represents the differentiation of $S_i(\pi)$ with respect to $\pi$. After a match is revealed to be good ($\pi=1$), the surplus in this case is expressed by
\[
S_1(1) = y_h - \gamma + (1 - \beta) d - \eta c \theta / \beta (1 - \theta),
\]
(15)

We assume that $y_h$ is high enough that the numerator of (15) is positive, which would indicate that (14) has a positive sign and that $S_i(\pi)$ is increasing in $\pi$.

We next consider the existence of $\pi_T$ that satisfies $S_0(\pi_T) = S_T(\pi_T)$. It follows from (9) and (11) that
\[
S_0(\pi) - S_T(\pi) = \beta (1 - \lambda) [a ^ \pi d + (1 - a) S(\pi)] - \beta d,
\]
and the condition $S_0(\pi_T) = S_T(\pi_T)$ is expressed as
\[
(1 - \lambda)(1 - a) S(\pi_T) = [1 - a \pi_T (1 - \lambda)] d,
\]
\[
\Rightarrow \frac{(1 - \lambda)(1 - a)}{1 - \lambda (1 - \lambda) (1 - a)} \left\{ y_h - \gamma + \beta (1 - \lambda) \alpha S(1) \pi_T + y_l - \gamma + (1 - \beta) d - \frac{\eta c \theta}{\beta (1 - \theta)} \right\}
\]
\[
= d - \alpha (1 - \lambda) d \pi_T.
\]
(17)

This results in
\[
\pi_T = \frac{\eta c \theta \beta (1 - \eta) - (y_l - \gamma) + [1 - (1 - \lambda)(1 - a)] d/(1 - \lambda)(1 - a)}{y_h - y_l + \beta \alpha (1 - \lambda) S(1) + \alpha [1 - \beta (1 - \lambda)(1 - a)] d/(1 - a)}.
\]
(17')

Subsequently, (11) yields the explicit form of $\pi_T$ that is characterised by $S_T(\pi_T) = 0$:
\[
\pi_T = \frac{\eta c \theta \beta (1 - \eta) - (y_l - \gamma)}{y_h - y_l + \beta \alpha (1 - \lambda) S(1) - d}.
\]
(18)

Because $\pi_T$ must be greater than zero, we suppose that $y_l$ satisfies the following condition for a given $\theta$:
\[
\frac{\eta c \theta}{\beta (1 - \eta)} - (y_l - \gamma) > 0.
\]
(19)

This condition requires that $y_l$ should be sufficiently low for any positive $\theta$. Otherwise, even if the true productivity is $y_l$ with probability one, the expected costs of recruiting job seekers are smaller than the expected net productivity $(y_l - \gamma)$, and every worker–firm pair yields non-negative profits, regardless of the realisation of $\pi$. We do not consider this case.

For the value of a $\pi_T$ that is a solution to equation (17), the following proposition is obtained.

**Proposition 1**

There exists a unique $\pi_T$ that satisfies (17) and is contained in the interval $(0, 1]$ if (19) holds and $y_h$ is sufficiently high to satisfy the following condition for a given $\theta$:
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\[ y_h - \gamma - \frac{\eta c \theta}{\beta(1-\eta)} \frac{\lambda[1-\beta \phi(1-\lambda)]d}{(1-\lambda)(1-a)} \geq 0. \]  

(20)

Furthermore, \( \pi_p \) is strictly greater than \( \pi_r \).

Proof

See Appendix A.

We permit the case \( \pi_p = 1 \), as there are countries in which the hiring threshold for permanent jobs is quite high, and most newly created jobs are temporary.\(^{12}\)

Employers set a higher hiring threshold for permanent contracts relative to temporary contracts. The crucial difference between the two types of contract is whether dismissal costs are imposed. If \( \pi \) takes a low value, a contract is more likely to be terminated. In a permanent contract, an employer must pay these costs for termination, and a lower \( \pi \) increases the probability of this event. Thus, a high probability of a good match is necessary for a permanent contract to compensate for higher expected costs. We further note that the employers’ choice between permanent and temporary contracts is endogenous and that a unique value of \( \pi_p \) exists. Given the assumption that temporary jobs last one period, a portion of newly created jobs take the form of permanent contracts (\( \pi_p < 1 \)).

3.2 Job creation

The measure of vacant jobs that is posted in equilibrium is determined by the free-entry condition. Equation (4), with \( V=0 \), implies

\[ \frac{c}{(1-\eta)q(\theta)} = \beta \left[ \int_{\pi_r}^{\pi_p} S_\pi(\pi) dG(\pi) + \int_{\pi_p}^{1} S_0(\pi) dG(\pi) \right], \]

\[ = \beta \left[ S'_1 \int_{\pi_r}^{\pi_p} (1-G(\pi)) d\pi + S'_0 \int_{\pi_p}^{1} (1-G(\pi)) d\pi \right], \]

(21)

where

\[ S'_{\pi} = y_h - y_t + \beta(1-\lambda) a [S(1) - d], \quad S'_0 = S'_1 = \frac{y_h-y_t + \beta(1-\lambda) a S(1)}{1-\beta(1-\lambda)(1-a)}. \]

The LHS of (21) increases in \( \theta \) because \( q(\theta) \) is a decreasing function of \( \theta \). The impact of \( \theta \) on the RHS of (21) is more complicated, but we can show that it decreases in \( \theta \). Differentiating the RHS of (21) with respect to \( \theta \) yields

\(^{12}\) According to Güell and Petrongolo (2007), the share of fixed-term contracts in new hires reached approximately 91–95 per cent during 1985–2002 in Spain. This tendency is also observed in other European countries in which restrictions on the use of fixed-term contracts have been relaxed, as in France.
\[
\beta \left[ \frac{dS_I}{d\theta} \int_{\pi}^{\pi^*} (1-G(\pi)) d\pi - (1-G(\pi^*)) [S_i - S_I] \frac{d\pi^*}{d\theta} \right.
\]

\[
- S_I (1-G(\pi^*)) \frac{d\pi^*}{d\theta} + \frac{dS_i}{d\theta} \int_{\pi}^{1} (1-G(\pi)) d\pi \right].
\]

From (15), \(S(1)\) decreases in \(\theta\), and the following results hold:

\[
d\pi^*/d\theta > 0, \quad d\pi^*/d\theta > 0, \quad dS_i/d\theta < 0, \quad dS^*/d\theta < 0.
\]

The first two results show that increased market tightness reduces the meeting probability of employers; hence, they raise their hiring threshold to ensure profits. Taking into account, from (16), that \(S_I > S^*_I\) for any \(\pi\), we conclude that the RHS of (21) decreases with \(\theta\). The conclusion of this subsection can thus be summarised as follows:

**Proposition 2**

*There exists a unique value of \(\theta\) that satisfies the job creation condition (21).*\(^{13}\)

As observed by Bucher (2010), it is difficult to show analytically the unique existence of a steady-state equilibrium in a Pries–Rogerson type model in which permanent and temporary jobs are incorporated. However, our model overcomes this difficulty and enables us to investigate the impact of employment protection analytically.

### 3.3 Employment flows

Let us denote the steady-state measure of permanent workers by \(e_p\) and the measure of temporary workers by \(e_t\). Similarly, the measure of matches that are known to be good is denoted by \(e_g\), and the measure of matches that have unknown quality is denoted by \(e_n\). In the steady-state equilibrium, the following equivalence conditions must hold in each employment pool.

The equivalence of the inflow and outflow from the employment pool of temporary contracts yields the following condition:

\[
e_t = \theta q(\theta) [G(\pi_t) - G(\pi^*_t)] u.
\]

The LHS of (22) reflects the assumption that every temporary contract is terminated in the next period, and each temporary worker will either be employed with a permanent contract.

\(^{13}\) A boundary condition at \(\theta=0\) is necessary for the existence of a unique intersection between the locus of the LHS of (21) and that of the RHS of (21). As \(q(\theta)\) goes to one as \(\theta\) goes to zero, either high recruiting costs or strong bargaining power of workers must be present to obtain the conclusion that the locus of the LHS of (21) lies below its RHS.
or unemployed. Regarding the RHS of (22), only worker–firm pairs that realise a signal contained in \([\pi_r, \pi_f]\) form matches in the form of temporary contracts.

In the pool of high quality employment, equivalence of inflows and outflows yields

\[
\lambda e_o = (1-\lambda) \alpha \left[ \frac{1}{G(\pi_r)} - \frac{1}{G(\pi_f)} \int_{\pi_r}^{\pi_f} \pi dG(\pi) \right],
\]

where

\[
E[\pi|\pi_t \leq \pi < \pi_r] = \frac{1}{G(\pi_r)} - \frac{1}{G(\pi_f)} \int_{\pi_r}^{\pi_f} \pi dG(\pi),
\]

\[
E[\pi|\pi_r \leq \pi] = \frac{1}{1-G(\pi_r)} \int_{\pi_r}^{\pi} \pi dG(\pi).
\]

The LHS of (23) indicates that a negative shock, which occurs with probability \(\lambda\), causes the separation of the employment relationship. The RHS of (23) indicates that matches for both temporary positions and permanent positions with unknown type are revealed to be high productivity with probability \((1-\lambda)\alpha\) multiplied by the expected value of the signal \((\alpha \pi)\) represents the probability that an unknown match with signal \(\pi\) turns out to be good.

In the employment pool of permanent jobs with unknown productivity, the following equivalence condition is obtained:

\[
[\lambda + (1-\lambda) \alpha] (e_o - e_f) = \theta q(\theta) \left[ 1 - G(\pi_r) \right] u.
\]

In the LHS of (26), outflows from this employment pool result from negative economic shocks and the revelation of matches that are either low or high productivity. The RHS of (26) captures inflows into this pool; it is composed of newly formed matches with signals that exceed \(\pi_r\). Note that \(e_f - e_o (= e_o - e_f)\) represents the measure of matches with unknown productivity in permanent contracts.

Together with two additional conditions

\[
e_n = e_f + e_f - e_o,
\]

\[
u = 1 - e_f - e_f,
\]

(22), (23) and (26)–(28) determine the steady-state value of \(e_o, e_f, e_o, e_n\) and \(u\). Solving these equations, we obtain

\[
u = \frac{\lambda + (1-\lambda) \alpha}{\Phi(\theta, \pi_r, \pi_f)},
\]

\[
e_o = \theta q(\theta) \left[ \frac{(1-\lambda)G(\pi_f)}{\lambda + (1-\lambda) \alpha} \right] + \frac{(1-\lambda)G(\pi_f)}{\lambda + G(\pi_f)} \left[ G(\pi_f) - G(\pi_r) \right] u,
\]

\[
e_f = \theta q(\theta) (G(\pi_f) - G(\pi_r)) u,
\]
Figure 1 Employment flows \((e^e\) is the measure of permanent workers with unknown match productivity)

\[
e_e = \frac{(1-\lambda)a\theta q(\theta)u}{\lambda} \left\{ \frac{(1-G(\pi_r))E[\pi|\pi_r \leq \pi]}{\lambda+(1-\lambda)a} \right\} 
+ (G(\pi_r) - G(\pi_t))E[\pi|\pi_r \leq \pi < \pi_r], \tag{32}
\]

\[
e_n = \frac{\theta q(\theta)[\lambda+(1-\lambda)a](G(\pi_r) - G(\pi_t))}{\lambda+(1-\lambda)a}u, \tag{33}
\]

where

\[
\Phi(\theta, \pi_r, \pi_t) = \lambda[\lambda+(1-\lambda)a] + \theta q(\theta)[\lambda+(1-\lambda)a](G(\pi_r) - G(\pi_t)) 
\times (\lambda+(1-\lambda)aE[\pi|\pi_r \leq \pi < \pi_r]) + (1-G(\pi_r))(\lambda+(1-\lambda)aE[\pi|\pi_r \leq \pi]).
\]

3.4 Wages

As the surplus sharing rules have already been derived, we can solve the Nash wage equations for each type of employment contract. From (4) and (8), the wage equations for temporary contracts are given by

\[
w_e(\pi) = \gamma + (1-\beta)U + \eta S_\tau(\pi) - \beta(1-\lambda)a\pi \eta S_\delta(1). \tag{34}
\]

Similarly, it follows from (5) and (7) that the wage equations for permanent contracts of type \(I\) are given by

\[
w_\delta(\pi) = \gamma + (1-\beta)U + \eta S_\delta(\pi) - \beta(1-\lambda)(a\pi S_\delta(1) + (1-a)S_\delta(1)), \tag{35}
\]

\[
w_\tau(\pi) = \gamma + (1-\beta)U + \eta S_\tau(\pi) - \beta(1-\lambda)(a\pi S_\tau(1) + (1-a)S_\tau(1)). \tag{36}
\]

The steady-state equilibrium in this model is characterised by \(\{\pi_r, \pi_t, \theta, u, e_r, e_t, e_o, e_n, w_e(\pi), w_\delta(\pi)(I=0,1)\}\), which are determined by (17'), (18), (21), (29)—(36). Because \(\theta, \pi_r\) and \(\pi_t\) are uniquely determined, other endogenous variables are also uniquely determined.

4. The effect of dismissal costs on hiring decisions

4.1 The direct effect of dismissal costs

To examine the effects of dismissal costs on firms’ hiring decisions in steady state equilibrium, we first consider how these costs affect labour market tightness. For that
purpose, the effects of $d$ on the hiring thresholds must be identified. It follows from (15) and (18) that, for a given $\theta$, a response of $\pi_\ell$ to a change in $d$ is given by

$$\frac{\partial \pi_\ell}{\partial d} = \frac{\gamma c\theta}{\beta(1-\eta)} - (y_\ell - \gamma) \right] \left[ \frac{\beta^2 \alpha(1-\lambda)}{(1-\beta(1-\lambda))(y_h - y_\ell + \beta\alpha(1-\lambda)(S_\ell(1-d)))^2} > 0. \quad (37) $$

A higher dismissal cost raises the hiring threshold for temporary jobs because an increase in the future expected surplus is offset by the increased firing cost; employers tend to cover this cost by hiring more high-potential employees (that is, workers with higher $\pi$).

From (17'), a change in $d$ has the following effect on $\pi_\ell$ for a given $\theta$:

$$\frac{\partial \pi_\ell}{\partial d} = \left[ y_h - y_\ell + \beta\alpha(1-\lambda)S_\ell(1-d) + \frac{Y_c\theta}{1-\beta(1-\lambda)} \right] \left[ y_h - y_\ell + \beta\alpha(1-\lambda)(S_\ell(1-d))^2 \right] \right\} \left[ \frac{\beta(1-\lambda)}{(1-\beta(1-\lambda))(y_h - y_\ell + \beta\alpha(1-\lambda)(S_\ell(1-d)))} \right],$$

where

$$\chi = \frac{\alpha(1-\beta(1-\lambda)(1-a))(1-a)}{1-a} \quad \text{and} \quad \nu = \frac{1-(1-\lambda)(1-a)}{(1-\lambda)(1-a)}.$$

Although (38) is somewhat complex, we can identify its sign explicitly. The result is summarised in the following lemma.

**Lemma 1**

An increase in dismissal costs raises $\pi_\ell$ for a given $\theta$ if (20) is satisfied.

**Proof**

See Appendix B.

We can provide the same interpretation as in the case of $\pi_\ell$. Using the results regarding the hiring thresholds described above, the effect of an increased dismissal cost on job creation can be examined. The effect of an increase in $d$ on the RHS of (21) for a given $\theta$ is given as follows:

$$\beta \left( \int_{S_\ell}^{S_r} (1-G(\pi)) d\pi - (S_r-S_\ell)(1-G(\pi_\ell)) \frac{\partial \pi_\ell}{\partial d} \right. - S_r(1-G(\pi_\ell)) \frac{\partial \pi_\ell}{\partial d} + \int_{S_\ell}^{S_r} (1-G(\pi)) d\pi \right).$$

To identify the sign of (39), we note that the following lemma holds.
Lemma 2

For a given $\theta$ and a sufficiently high $\beta$, $\pi_r - \pi_t$ is increasing in $d$ if $y_h$ satisfies

$$y_h - \gamma - \frac{\eta c \beta}{\beta (1 - \eta)} - 2\beta \lambda d > 0,$$

and $a$ satisfies

$$a \leq \frac{1 - 2\lambda}{2(1 - \lambda)},$$

(40)

The inequality (40) is likely to be satisfied for a lower $\lambda$.

Proof

See Appendix C.

From (37) and Lemma 1, we obtain the following proposition regarding the effect of dismissal costs on job creation.

Proposition 3

The sign of (39) is negative and $\partial \theta / \partial d < 0$ if $\beta$ is sufficiently high.

Proof

See Appendix D.

An increase in dismissal costs reduces the expected profits of firms with vacancies, leading to decreased job creation. The result obtained in Proposition 3 is standard and intuitive. However, the main focus of this paper lies in how the costs of firing employees affect employers’ hiring policies. Unfortunately, the effects of $d$ on $\pi_r$ and $\pi_t$ are both ambiguous when the effect through $\theta$ is taken into account. An increase in dismissal costs raises the hiring thresholds for both types of employment contract, while job creation is reduced as a result of this increase, giving rise to a decline in both thresholds. We will therefore investigate the effect of $d$ on the difference $\pi_r - \pi_t$.

Proposition 4

The difference in the hiring thresholds, $\pi_r - \pi_t$ is increasing in $d$ if $d$ is sufficiently large.

Proof

See Appendix E.
For higher \( d \), \( \pi_p - \pi_r \) increases, indicating that employers raise the hiring threshold more for permanent than for temporary employment. This implies that employers are more selective in hiring permanent workers than temporary workers. Specifically, screening employees through temporary employment becomes more important for employers because permanent contracts involve higher future dismissal costs. Furthermore, Propositions 3 and 4 have the following implication. Reducing dismissal costs not only increases job creation but reduces the difference \( \pi_p - \pi_r \). A reduction in the difference in hiring thresholds may lead to an increase in the relative proportion of permanent employment but a decrease in the average productivity of jobs, as employers may become less concerned about the quality of matches. Accordingly, lower dismissal costs may help generate new jobs in the economy, but we should be mindful of the effect of this institutional change on the average productivity of labour.

To obtain clear-cut results regarding the effects of a change in \( d \) on \( \pi_p \) and \( \pi_r \), we conduct the following numerical exercise. In accordance with common practice, we suppose that the matching function is Cobb-Douglas: \( m(u, v) = au^v v^{1-\ell} \). The distribution function for \( \pi \) is assumed to be uniform.\(^{14} \) The parameter values are chosen as described in Table 1. The values of most of these parameters are based on related work, while several parameters are chosen to make the numerical results reasonable.

Table 2 shows how a change in dismissal costs affects labour market tightness and the hiring threshold for each job type. It follows from (37) and Lemma 1 that an increase in \( d \) raises the hiring thresholds for both job types, for a given \( \theta \). According to the numerical exercise, a higher \( d \) raises the hiring threshold for permanent jobs, even allowing for the impact on \( \theta \) (as described in Proposition 3, a higher \( d \) decreases \( \theta \)), and reduces the threshold for temporary jobs. A lower \( \theta \), stemming from an increase in \( d \), raises the meeting probability for employers (\( q(\theta) \) is decreasing in \( \theta \)). Although this leads to a lower hiring threshold for each job type, it has a dominant impact only on temporary jobs. As employers with temporary employees do not pay dismissal costs, firing costs associated with permanent jobs affect the joint surplus in temporary jobs only indirectly, through their effect on the surplus in permanent jobs. Thus, the direct effect of \( d \) on \( \pi_r \) will not exceed the indirect effect through \( \theta \).

---

\(^{14} \) Following Pries (2004), Pries and Rogerson (2005), Albertini et al. (2009) and Faccini (2013) assume a mean-zero normal distribution with some positive standard deviation, truncated and rescaled in the support. Although using this type of distribution will enrich our numerical exercise, we place a high priority on determining the properties of firms’ hiring decisions in a simple framework.
4.2 The effect of $\alpha$

In this section, we examine how a change in the probability that the true quality of a match is revealed affects firms’ hiring decisions. It follows from the definition of $\alpha$ that a change in $\alpha$ results from a change in the range of the random productivity factor ($\varepsilon$). That is, the more volatile is productivity, the more difficult it is for employers to assess the true quality of a match. Uncertainty regarding productivity (or demand) is a major concern of employers and seriously affects hiring decisions. We therefore examine the effect of $\alpha$ on $\theta$, $\pi_r$ and $\pi_t$. As the hiring decision reduces to the determination of $\pi_r$ and $\pi_t$, we seek to determine the sign of $\partial \pi_t/\partial \alpha (j = P, T)$. It follows from (18) that an increase in $\alpha$ unambiguously reduces $\pi_t$ for a given $\theta$. If true match quality is revealed with higher probability, employers can set a lower hiring threshold, so that $S_r(\pi_t)=0$ is retained. Therefore, more efficient on-the-job screening lowers the hiring threshold for temporary workers. In the case of permanent workers, however, a different result is obtained as follows.

Proposition 5

**Suppose that $\alpha$ and $d$ are sufficiently high to satisfy**

\[
\beta \left[ a - (1-a)(a+\lambda(1-a)) \right] d - \beta (1-\lambda) \left[ -\frac{\eta \lambda}{\beta (1-\eta)} - (y_i - \gamma) \right] \geq 0, \quad (41)
\]

**and that the following condition holds:**

\[
y_h - \gamma - \frac{\eta \lambda}{\beta (1-\eta)} - \left[ \alpha \beta (1-\lambda) + \lambda \right] d > 0. \quad (42)
\]

**Then, an increase in $\alpha$ raises $\pi_r$ for a given $\theta$.**

**Proof**

15) We suppose that $\alpha$ satisfies the following condition: $a \geq (\lambda + \sqrt{\lambda})/(1-\lambda)$. This ensures the following inequality: $a - (1-a)(a+\lambda(1-a)) \geq 0$. 

<table>
<thead>
<tr>
<th>$d$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
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<td>1.867</td>
<td>1.832</td>
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<td>1.762</td>
<td>1.728</td>
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<td>1.628</td>
<td>1.596</td>
<td>1.563</td>
</tr>
<tr>
<td>$\pi_r$</td>
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<tr>
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<td>0.250</td>
<td>0.240</td>
<td>0.231</td>
<td>0.222</td>
<td>0.214</td>
</tr>
</tbody>
</table>

**Table 2 The effect of $d$ on $\theta$, $\pi_r$ and $\pi_t$**
See Appendix F.

Note that we have already shown that \( \partial S_T(\pi)/\partial a > 0 \) for any \( \pi \) and that \( \pi_r \) is decreasing in \( a \) for given \( \theta \). In contrast, the effect of \( a \) on \( \pi_r \) crucially depends on how \( a \) affects \( S_T(\pi) \) and \( S_0(\pi) \). If \( S_0(\pi) \) decreases or increases slightly when \( a \) increases, \( \pi_r \) increases. However, the impact of \( a \) on \( S_0(\pi) \) is not determinate, owing to the assumption that firms with permanent jobs retain their employees, even if the true quality of a match is unknown. In this model, more efficient on-the-job screening reduces the probability of retaining a current job, while it increases the expected profits of firms. For higher dismissal costs, the former effect exceeds the latter, and the sign of \( \partial S_0(\pi)/\partial a \) is likely to be negative: from (12) and (13), we obtain

\[
\frac{\partial S_0(\pi)}{\partial a} = -\frac{\beta(1-\lambda)(1-\pi)[y_1-\gamma + (1-\beta)d - \eta \theta \beta(1-\gamma)]}{[1-\beta(1-\lambda)(1-a)]^2},
\]

for a given \( \theta \). Thus, in an economy with high dismissal costs, the hiring threshold for permanent contracts will be raised by an increase in the probability that the true quality of match is revealed, while the hiring threshold for temporary contracts is lowered.

As \( \theta \) is fixed in the above analysis of the effect of \( a \) on \( \pi_r \) and \( \pi_T \), we numerically examine how a change in \( a \) affects \( \theta \), \( \pi_r \) and \( \pi_T \). Following Faccini (2013), the firing costs are fixed at \( d=5.16 \) in this calculation. Although it is difficult, theoretically, to determine the response of \( \theta \) to a change in \( a \), Table 3 shows that an increase in \( a \) increases \( \theta \). Recalling that, for a given \( \theta \), a rise in \( a \) reduces \( \pi_T \) and raises \( \pi_r \), the results obtained in Table 3 are noteworthy. Regarding the hiring threshold for temporary jobs, a direct effect of \( a \) on \( \pi_T \) exceeds its indirect effect through \( \theta \). Thus, a higher probability that the true match quality is revealed occasions a decrease in \( \pi_T \). Regarding the hiring threshold for permanent jobs, it follows from Table 3 that, for a smaller \( a \), \( \pi_T \) is decreasing in \( a \), while for a larger \( a \), \( \pi_r \) is increasing in \( a \). Furthermore, the sign of \( \partial \pi_T/\partial a \) changes with lower values of \( a \), as dismissal costs increase (see Table 4 and 5). When \( d=5.16 \), the value of \( \pi_T \) starts to increase at \( a=0.25 \). The corresponding values of \( a \), when \( d=1 \) and \( d=10 \), are \( a=0.45 \) and \( a=0.15 \), respectively. In short, a higher \( a \) leads to an increase in \( \pi_T \) in the broad area of \( a \), if dismissal costs are sufficiently high. This result is consistent with the statements of Proposition 4. The results obtained in this subsection indicate that the resolution of uncertainty with respect to the true match type does not always lead to a reduction of the hiring threshold for permanent jobs. As employers dismiss employees when the quality of matches is poor, requiring them to pay dismissal costs, higher dismissal costs cause the threshold to rise. The presence of dismissal costs will compel employers to be cautious in hiring workers for permanent positions.
Table 3 The effect of $\alpha$ on $\theta$, $\pi_F$ and $\pi_T$ when $d=5.16$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>0.05</th>
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<th>0.15</th>
<th>0.2</th>
<th>0.25</th>
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<td>1.758</td>
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<td>1.781</td>
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<td>1.793</td>
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<td>$\pi_F$</td>
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<td>0.433</td>
<td>0.429</td>
<td>0.432</td>
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Table 4 The effect of $\alpha$ on $\theta$, $\pi_F$ and $\pi_T$ when $d=1$

<table>
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<th>0.25</th>
<th>0.3</th>
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Table 5 The effect of $\alpha$ on $\theta$, $\pi_F$ and $\pi_T$ when $d=10$

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</table>

5. Conclusions

This paper has examined how employment protection of permanent contracts affects the hiring decisions of employers if the true productivity of a worker–firm pair is not fully revealed even after a match is formed. We incorporate permanent and temporary contracts into an equilibrium search model and consider a situation in which temporary contracts are used to screen workers for permanent positions. Although employers cannot accurately observe the true quality of a match, they receive an observable signal about the quality of the match when hiring a worker. Employers' hiring decisions are based on the realisation of this signal and are characterised as the determination of the hiring thresholds.

The main results obtained in this paper are summarised as follows. First, reducing dismissal costs increases job creation. Second, a reduction in dismissal costs entails a reduction in the difference in hiring thresholds between permanent and temporary contracts, which implies that employers will be less selective in hiring workers into permanent contracts when dismissal costs fall. From a different perspective, temporary contracts are likely to be used as screening devices in countries with stringent employment protection of permanent employment. Third, a rise in the probability that the true quality of a match is revealed lowers the hiring threshold for temporary contracts but raises the threshold for permanent contracts if employers' costs of dismissing permanent workers rise.
March 2014  Makoto MASUI

Although we performed numerical calculations to illustrate how some key parameters affect job creation and firms’ hiring decisions, the results obtained are not robust, as the choice of parameter values and functional forms is not based on calibration. To make the results of our model more convincing, we must examine whether similar results are obtained through a more sophisticated quantitative analysis.

Appendix

A. Proof of Proposition 1

We prove the statements of the proposition by relying on the following three facts: (i) \( S_0(0) \) is strictly less than \( S_T(0) \); (ii) \( S_0(1) \) is strictly greater than \( S_T(1) \); and (iii) \( S_0(\pi_T) \) is strictly less than zero, where \( \pi_T \) is given by (18).

First, it follows from (16) that \( S_0(0) - S_T(0) \) is expressed by

\[
S_0(0) - S_T(0) = \beta(1 - \lambda)(1 - a)S_0(0) - \beta d, \\
= \frac{\beta(1 - \lambda)(1 - a)}{1 - \beta(1 - \lambda)(1 - a)} \left[ y_1 - \gamma - \frac{yc\theta}{\beta(1 - \eta)} + (1 - \beta)d \right] - \beta d, \\
= \frac{\beta(1 - \lambda)(1 - a)}{1 - \beta(1 - \lambda)(1 - a)} \left[ y_1 - \gamma - \frac{yc\theta}{\beta(1 - \eta)} \right], \\
+ \frac{\beta[1 - \lambda](1 - a)(1 - \beta) - 1 + \beta(1 - \lambda)(1 - a)]d}{1 - \beta(1 - \lambda)(1 - a)} \\
= \frac{\beta(1 - \lambda)(1 - a)}{1 - \beta(1 - \lambda)(1 - a)} \left[ y_1 - \gamma - \frac{yc\theta}{\beta(1 - \eta)} \right] - \beta \frac{1 - \lambda(1 - a)}{1 - \beta(1 - \lambda)(1 - a)} d. \\
\]  

(A1)

If (19) is satisfied, the first term in (A1) and the total sign of (A1) are negative.

Second, from (11), we obtain

\[
S_0(1) - S_T(1) = [1 - \beta a(1 - \lambda)]S_0(1) + \frac{yc\theta}{\beta(1 - \eta)} - (y_h - \gamma). \\
\]  

(A2)

We note that from (13) and (15), \( S_0(1) \) is represented by

\[
S_0(1) = S_1(1) - d = \frac{y_h - \gamma - yc\theta[\beta(1 - \eta) + (1 - \beta)d - (1 + \beta\lambda)d}{1 - \beta(1 - \lambda)} \\
= \frac{y_h - \gamma - yc\theta[\beta(1 - \eta) - \beta\lambda d}{1 - \beta(1 - \lambda)}. \\
\]  

(A3)

Substituting (A3) into (A2) and arranging it yield

\[
S_0(1) - S_T(1) = \frac{1}{1 - \beta(1 - \lambda)} \left[ 1 - \beta a(1 - \lambda) \right] \left[ y_h - \gamma - \frac{yc\theta}{\beta(1 - \eta)} - \beta\lambda d \right] \\
+ \left[ 1 - \beta(1 - \lambda) \right] \left[ \frac{yc\theta}{\beta(1 - \eta)} - (y_h - \gamma) \right],
\]
\[
\frac{1}{1-\beta(1-\lambda)} \left\{ \beta(1-\lambda) - \beta \alpha(1-\lambda) \left( y_h - \gamma - \frac{\eta c \theta}{\beta(1-\eta)} \right) \right\} - \beta \lambda \left[ 1 - \beta \alpha(1-\lambda) \right] d, \\
\frac{1}{1-\beta(1-\lambda)} \left\{ y_h - \gamma - \frac{\eta c \theta}{\beta(1-\eta)} \right\} - \frac{\lambda}{1-\lambda} \left[ 1 - \beta \alpha(1-\lambda) \right] d.
\]

(A4)

This takes a non-negative value if (20) holds. Since both \(S_\ell(\pi)\) and \(S_T(\pi)\) are linear in \(\pi\), there exists a unique \(\pi_r \in (0, 1)\) that satisfies (17).

Finally, we will show that \(\pi_r\) is strictly greater than \(\pi_T\). To prove this statement, it is sufficient to show that \(S_\ell(\pi_r)\) has a negative sign because the slope of \(S_\ell(\pi)\) is larger than that of \(S_T(\pi)\) for any \(\pi\). Evaluating (9) by \(\pi_r\) and substituting (18) result in

\[
S_\ell(\pi_r) = [y_h - y_i + \beta(1-\lambda)S_1(1)] \pi_r + y_i - \gamma + \beta(1-\lambda)(1-a)[S_\ell(\pi_r) + d] - \frac{\eta c \theta}{\beta(1-\eta)} - \beta d,
\]

and

\[
[1 - \beta(1-\lambda)(1-a)] S_\ell(\pi_r) = \frac{y_h - y_i + \beta(1-\lambda)S_1(1)}{y_h - y_i + \beta(1-\lambda)S_1(1) - \beta(1-\lambda)ad} d,
\]

\[
\Rightarrow [1 - \beta(1-\lambda)(1-a)] S_\ell(\pi_r) = \frac{y_h - y_i + \beta(1-\lambda)S_1(1)}{y_h - y_i + \beta(1-\lambda)S_1(1) - \beta(1-\lambda)ad} d,
\]

\[
\times \left[ \frac{\eta c \theta}{\beta(1-\eta)} - (y_i - \gamma) \right] - \left[ \frac{\eta c \theta}{\beta(1-\eta)} - (y_i - \gamma) \right] - \beta(1-\lambda) \left[ 1 - \beta(1-\lambda)(1-a) \right] d,
\]

\[
\Rightarrow \beta(1-\lambda) \left[ y_h - y_i + \beta(1-\lambda)S_1(1) - \beta(1-\lambda)ad \right] \left[ \frac{\eta c \theta}{\beta(1-\eta)} - (y_i - \gamma) \right] - \beta \lambda \left[ 1 - \beta \alpha(1-\lambda) \right] d,
\]

\[
\Rightarrow \beta(1-\lambda) \left[ y_h - y_i + \beta(1-\lambda)S_1(1) - \beta(1-\lambda)ad \right] \left[ \frac{\eta c \theta}{\beta(1-\eta)} - (y_i - \gamma) \right] - \beta \lambda \left[ 1 - \beta \alpha(1-\lambda) \right] d.
\]

(A5)

Regarding a sign of (A5), its first term in the brace is less than one because

\[
y_h - y_i + \beta(1-\lambda)S_1(1) - \beta(1-\lambda)ad - \left[ \frac{\eta c \theta}{\beta(1-\eta)} - (y_i - \gamma) \right],
\]

\[
y_h - y_i + \beta(1-\lambda)S_1(1) - \beta(1-\lambda)ad - \frac{\eta c \theta}{\beta(1-\eta)},
\]

\[
y_h - y_i + \beta(1-\lambda)S_1(1) - \beta(1-\lambda)ad - \frac{\eta c \theta}{\beta(1-\eta)} + \beta(1-\lambda)S_1(1) > 0,
\]

where condition (20) ensures that \(y_h - y_i + \beta(1-\lambda)S_1(1) - \beta(1-\lambda)ad > \frac{\eta c \theta}{\beta(1-\eta)}\).

Because the second term of the brace in (A5) is obviously greater than one, the overall sign of (A5) is negative. This indicates that \(S_\ell(\pi)\) is less than \(S_T(\pi)\) at \(\pi_r(S_T(\pi_r)=0\). The proof is complete.

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16) This fact is obtained by differentiating (16) with respect to \(\pi\) and using \(S'_i(\pi)>0\).
17) If (20) is satisfied, (A3) takes a positive value and therefore \(S_\ell(1)>0\).
B. Proof of Lemma 1

We first show that the coefficient of the last term in (38) is less than one. Actually, we obtain

\[ 1 - \frac{a(1-\lambda)[1-\beta(1-\lambda)(1+\beta(1-a))]}{(1-\beta(1-\lambda))(1-(1-\lambda)(1-a))} = \frac{\lambda[1-\beta(1-\lambda)] + \beta^2 a(1-a)(1-\lambda)^2}{(1-\beta(1-\lambda))(1-(1-\lambda)(1-a))} > 0. \]

We also note that the following inequality derives from (20):

\[ y_h - \gamma - \frac{\eta \sigma \theta}{\beta(1-\eta)} > 0. \]  

(B-1)

Then the sum of the first and the last terms in the brace of (38) has a positive sign and the overall sign of (38) is positive. The proof is complete.

C. Proof of Lemma 2

It follows from (17) and (18) that the numerator of \( \pi_d - \pi_f \) is expressed by

\begin{align*}
(y_h - y_i + \beta a(1-\lambda)S_i(1) - \beta a(1-\lambda)d & \left[ \frac{\eta \sigma \theta}{\beta(1-\eta)} - (y_i - \gamma) + \nu d \right] \\
- (y_h - y_i + \beta a(1-\lambda)S_i(1)) & \left[ \frac{\eta \sigma \theta}{\beta(1-\eta)} - (y_i - \gamma) \right], \\
= - (\beta a(1-\lambda) + \nu) & \left[ \frac{\eta \sigma \theta}{\beta(1-\eta)} - (y_i - \gamma) \right] d + \nu(y_h - y_i + \beta a(1-\lambda)S_i(1) - \beta a(1-\lambda)d) d, \\
= - \left[ \beta a(1-\lambda) + \frac{a[1-\beta(1-\lambda)(1-a)]}{1-a} \right] & \left[ \frac{\eta \sigma \theta}{\beta(1-\eta)} - (y_i - \gamma) \right] d \\
+ \nu(y_h - y_i + \beta a(1-\lambda)S_i(1) - \beta a(1-\lambda)d) d, \\
= - \frac{\alpha}{1-a} & \left[ \frac{\eta \sigma \theta}{\beta(1-\eta)} - (y_i - \gamma) \right] d + \nu[y_h - y_i + \beta a(1-\lambda)S_i(1) - \beta a(1-\lambda)d] d, \quad (C-1)
\end{align*}

Partially differentiating (C-1) with respect to \( d \) for a given \( \theta \) yields

\begin{align*}
\nu[y_h - y_i + \beta a(1-\lambda)S_i(1) - \beta a(1-\lambda)d] + & \nu \beta a(1-\lambda) \left( \frac{\partial S_i(1)}{\partial d} - 1 \right) d \\
- & \frac{\alpha}{1-a} \left[ \frac{\eta \sigma \theta}{\beta(1-\eta)} - (y_i - \gamma) \right], \\
= & \nu \left[ y_h - y_i + \beta a(1-\lambda)S_i(1) - \beta a(1-\lambda) \left[ 1 + \frac{\beta a}{1-\beta(1-\lambda)} \right] \right] d \\
- & \frac{\alpha}{1-a} \left[ \frac{\eta \sigma \theta}{\beta(1-\eta)} - (y_i - \gamma) \right], \\
= & \nu \left[ y_h - y_i + \beta a(1-\lambda) \left( y_h - y_i - \frac{\eta \sigma \theta \beta}{\beta(1-\eta)} + (1-\beta)d - \frac{(1-\beta + 2\beta \lambda)d}{1-\beta(1-\lambda)} \right) \right] \\
- & \frac{\alpha}{1-a} \left[ \frac{\eta \sigma \theta}{\beta(1-\eta)} - (y_i - \gamma) \right], \\
= & \nu \left[ y_h - y_i + \beta a(1-\lambda) \left( y_h - y_i - \frac{\eta \sigma \theta \beta}{\beta(1-\eta)} - \frac{2\beta \lambda d}{1-\beta(1-\lambda)} \right) - \frac{\alpha}{\nu(1-a)} \left[ \frac{\eta \sigma \theta}{\beta(1-\eta)} - (y_i - \gamma) \right] \right].
\end{align*}
\[
= \nu \left( y_h - y_t - \frac{\theta(1 - \lambda)}{1 - (1 - \lambda)(1 - \omega)} \left( \frac{\eta \theta}{\beta(1 - \eta)}(y_t - \gamma) \right) \right) \\
+ \beta \alpha(1 - \lambda) \left[ y_h - \gamma - \frac{\eta \theta}{\beta(1 - \eta)}(y_t - \gamma) - 2\beta \delta d \right].
\] (C-2)

The first line of (C-2) is positive because (20) holds and
\[
\frac{\alpha(1 - \lambda)}{1 - (1 - \lambda)(1 - \omega)} < 1.
\]
The second line of (C-2) also becomes positive if the following condition holds:
\[
y_h - \gamma - \frac{\eta \theta}{\beta(1 - \eta)} - 2\beta \delta d > 0.
\]
Whether this condition is more restrictive than (20) or not depends on the value of parameters. In any case, (C-2) takes a positive sign if \( y_h \) is sufficiently large.

We next examine how the denominator of \( \pi_p - \pi_r \) responds to a change in \( d \). The denominator of \( \pi_p - \pi_r \) is given by
\[
[y_h - y_t + \beta \alpha(1 - \lambda)S_t(1) - \beta \alpha(1 - \lambda)d][y_h - y_t + \beta \alpha(1 - \lambda)S_t(1) + \kappa d],
\]
\[
= (y_h - y_t + \beta \alpha(1 - \lambda)S_t(1))^2 + (y_h - y_t + \beta \alpha(1 - \lambda)S_t(1))[x - \beta \alpha(1 - \lambda)]d \\
- \kappa \beta \alpha(1 - \lambda)d^2.
\] (C-3)

Partially differentiating (C-3) with respect to \( d \) yields
\[
2\beta \alpha(1 - \lambda)(y_h - y_t + \beta \alpha(1 - \lambda)S_t(1)) \frac{\partial S_t(1)}{\partial d} + \beta \alpha(1 - \lambda)[x - \beta \alpha(1 - \lambda)]d \frac{\partial S_t(1)}{\partial d} \\
+ (y_h - y_t + \beta \alpha(1 - \lambda)S_t(1))[x - \beta \alpha(1 - \lambda)] - 2\kappa \beta \alpha(1 - \lambda),
\]
\[
= (y_h - y_t + \beta \alpha(1 - \lambda)S_t(1)) \left[ \frac{2\beta \alpha(1 - \lambda)(1 - \beta)}{1 - \beta(1 - \lambda)} + x - \beta \alpha(1 - \lambda) \right] \\
+ \beta \alpha(1 - \lambda)[x - \beta \alpha(1 - \lambda)]d \frac{\partial S_t(1)}{\partial d} - 2\kappa \beta \alpha(1 - \lambda).
\] (C-4)

Noticing that \( \partial S_t(1)/\partial d \) tends to be zero as \( \beta \to 1 \), (C-4) takes a negative sign if \( x - \alpha(1 - \lambda) \leq 0 \) or
\[
\alpha \leq \frac{1 - 2\lambda}{2(1 - \lambda)}. \hspace{1cm} (C-5)
\]

Based on the results of the above calculations, an increase in \( d \) decreases the denominator of \( \pi_p - \pi_r \) and increases its numerator. This means that \( \pi_p - \pi_r \) is increasing with respect to \( d \) for a given \( \theta \). The proof is complete.

**D. Proof of Proposition 3**

The second term of (39) takes a negative value from Lemma 1 and \( S_0 - S_t > 0 \). (37) ensures that a sign of the third term of (39) is negative. Although the last term is positive
because $\partial S'_\theta / \partial d > 0$, this becomes quite small for a sufficiently high $\beta$ because
\[
\frac{\partial S'_\theta}{\partial d} = \frac{\alpha \beta (1-\lambda)(1-\beta)}{(1-\beta(1-\lambda))[1-\beta(1-\lambda)(1-\alpha)]].
\]
Thus the overall sign of (39) is likely to be negative ($\partial S'_\theta / \partial d < 0$).

Remembering that the LHS of (21) is increasing and the RHS of (21) is decreasing in $\theta$, we finally find that an increase in $d$ decreases $\theta$. The proof is complete.

E. Proof of Proposition 4

We have already shown in Lemma 2 that a higher $d$ increases $\pi_p - \pi_T$ for a given $\theta$. Now we will examine how a change in $\theta$ affects the difference in the hiring thresholds.

Before conducting the main calculation, some preliminary results are derived for any $\theta$:
\[
\frac{d}{d\theta} \text{ the denominator of } \frac{\pi_p - \pi_T}{d} \quad \text{is} \quad \frac{d}{d\theta} \{ \frac{\eta c d}{\beta(1-\eta)} \left[ \frac{\alpha}{1-\alpha} \right] + \frac{\nu \beta a(1-\lambda)}{1-\beta(1-\lambda)} \}.
\]

\[
\frac{d}{d\theta} \text{ the numerator of } \frac{\pi_p - \pi_T}{d} \quad \text{is} \quad \frac{d}{d\theta} \left\{ \frac{\eta c d}{\beta(1-\eta)} \left[ \frac{\alpha}{1-\alpha} + \frac{\nu \beta a(1-\lambda)}{1-\beta(1-\lambda)} \right] \right\}.
\]

where we define $\phi$ as $\phi = y_\theta - y_\theta + \beta a(1-\lambda) S_\theta(1)$.

Now we compute $d(\pi_p - \pi_T)/d\theta$. Then it follows from (E-1) and (E-2) that the numerator of $d(\pi_p - \pi_T)/d\theta$ is written by
\[
\frac{\eta c d}{\beta(1-\eta)} \left[ \frac{\alpha}{1-\alpha} + \frac{\nu \beta a(1-\lambda)}{1-\beta(1-\lambda)} \right] \left\{ \frac{\alpha}{1-\alpha} + \frac{\nu \beta a(1-\lambda)}{1-\beta(1-\lambda)} \right\} \times \left\{ \frac{\eta c d}{\beta(1-\eta)} \left[ \frac{\alpha}{1-\alpha} + \frac{\nu \beta a(1-\lambda)}{1-\beta(1-\lambda)} \right] \right\}.
\]
A sign of entire (E-3) depends on a sign of the first term in (E-3). To identify the sign of the brace in the first term, it is sufficient to focus on the following:

\[
\frac{a(\phi + xd)}{1-a} + \nu \beta a(1-\lambda)(\phi + xd) - \nu \beta a(1-\lambda)(2\phi + (x - \beta a(1-\lambda)) d) \quad (E-4)
\]

By the definition of \( \phi \), (E-4) increases with respect to \( d \) for a given \( \theta \). This means that (E-3) is likely to take a negative value for a higher \( d \) and that \( \pi_p - \pi_r \) decreases with respect to \( \theta \). From Proposition 3, we have already known \( \partial \theta/\partial d < 0 \) in the steady-state equilibrium. Thus, the indirect effect of \( d \) on \( \pi_p - \pi_r \) through \( \theta \) is positive.

Combining the result of Lemma 2, we conclude that the total effects of dismissal costs on the difference in the hiring thresholds are negative. The proof is complete.

F. Proof of Proposition 5

From (17'), we obtain

\[
\begin{align*}
\frac{\partial \pi_p}{\partial a} &= \frac{1}{[y_h - y_i + \beta a(1-\lambda)S_i(1) + xd]^2} \left[ \frac{[y_h - y_i + \beta a(1-\lambda)S_i(1) + xd]d}{(1-\lambda)(1-a)} ight. \\
&\quad \left. - \left[ \frac{\nu c d}{\beta(1-\gamma)} - (y_i - \gamma) + \nu d \right] \left[ \beta(1-\lambda)S_i(1) + \frac{a + (1-a)[1 - \beta(1-\lambda)(1-a)]d}{(1-a)^2} \right] \right],
\end{align*}
\]

where for a given \( \theta \),

\[
\begin{align*}
\frac{\partial \nu}{\partial a} &= \frac{(1-\lambda)(1-a) + (1-\lambda)(1-a)}{(1-\lambda)(1-a)^2} = \frac{1}{(1-\lambda)(1-a)^2} > 0, \\
\frac{\partial \kappa}{\partial a} &= \frac{[1 - \beta(1-\lambda)(1-a) + \beta(1-\lambda)(1-a)](1-a) + a[1 - \beta(1-\lambda)(1-a)]}{(1-a)^2}, \\
&= \frac{(1-a)[1 - \beta(1-\lambda)(1-a)] + a}{(1-a)^2} > 0.
\end{align*}
\]

\(^{18}\) We also notice that

\[
\frac{\partial}{\partial d}(\phi - \beta a(1-\lambda)d) = \beta a(1-\lambda) \left( \frac{\partial S_i(1)}{\partial d} - 1 \right),
\]

where \( \partial \phi/\partial d = \beta a(1-\lambda)S_i(1)/\partial d \).
Regarding the numerator of (F-1), we focus on the coefficients of \( S_i(1) \), which are given by

\[
\begin{align*}
\frac{\beta a d}{(1-a)^2} & \beta(1-\lambda) \left[ \frac{\eta c \theta}{\beta(1-\eta)} - (y_i - \gamma) + \nu d \right], \\
= & \beta a d (1-a)^2 - \beta (1-(1-\lambda)(1-a)) d - \beta(1-\lambda) \left[ \frac{\eta c \theta}{\beta(1-\eta)} - (y_i - \gamma) \right], \\
= & \beta \left[ a - (1-a)(1-(1-\lambda)(1-a)) \right] d - \beta(1-\lambda) \left[ \frac{\eta c \theta}{\beta(1-\eta)} - (y_i - \gamma) \right].
\end{align*}
\]

\( \text{(F-2)} \)

The first term of (F-2) is non-negative if \( a - (1-a)(1-(1-\lambda)(1-a)) \geq 0 \). This result is obtained when \( a \) satisfies

\[ a \geq -\lambda + \sqrt{\lambda}. \]

\( \text{(F-3)} \)

For a higher \( d \), (F-2) is likely to be positive under (F-3) because the equilibrium value of \( \theta \) is a decreasing function of \( d \) as shown in Proposition 3.

We next examine a sign of the other terms in the numerator of (F-1). That is,

\[
\begin{align*}
\frac{1}{(1-\lambda)(1-a)^2} \left[ y_n - y_i + a(1-\beta(1-\lambda)(1-a)) \right] d - \beta(1-\lambda) \left[ \frac{\eta c \theta}{\beta(1-\eta)} - (y_i - \gamma) \right] \\
+ \frac{(1-(1-\lambda)(1-a)) d}{(1-\lambda)(1-a)} \left[ (1-a)(1-\beta(1-\lambda)(1-a)) + a \right] d.
\end{align*}
\]

\( \text{(F-4)} \)

Arranging (F-4) yields

\[
\begin{align*}
\frac{d}{(1-\lambda)(1-a)^2} \left[ y_n - y_i - (1-\lambda)(1-a)(1-\beta(1-\lambda)(1-a)) + a \right] \\
\times \left[ \frac{\eta c \theta}{\beta(1-\eta)} - (y_i - \gamma) \right] + \frac{a(1-\beta(1-\lambda)(1-a)) d}{1-a} \\
- \frac{(1-(1-\lambda)(1-a)) d}{(1-\lambda)(1-a)} \left[ (1-a)(1-\beta(1-\lambda)(1-a)) + a \right] d.
\end{align*}
\]

\[
\geq \frac{d}{(1-\lambda)(1-a)^2} \left[ y_n - y_i - \frac{\eta c \theta}{\beta(1-\eta)} - (y_i - \gamma) \right] + \frac{a(1-\beta(1-\lambda)(1-a)) d}{1-a} \\
- \frac{(1-(1-\lambda)(1-a)) d}{(1-\lambda)(1-a)} \left[ (1-a)(1-\beta(1-\lambda)(1-a)) + a \right] d.
\]

\[
\geq \frac{d}{(1-\lambda)(1-a)^2} \left[ y_n - y_i - \frac{\eta c \theta}{\beta(1-\eta)} - (y_i - \gamma) \right] + \frac{\alpha(1-\beta(1-\lambda)(1-a)) d}{1-a} \\
- \frac{(1-(1-\lambda)(1-a)) d}{(1-\lambda)(1-a)} \left[ (1-a)(1-\beta(1-\lambda)(1-a)) + a \right] d.
\]

\[
= \frac{d}{(1-\lambda)(1-a)^2} \left[ y_n - y_i - \frac{\eta c \theta}{\beta(1-\eta)} - (y_i - \gamma) \right] + \frac{\alpha(1-\beta(1-\lambda)(1-a)) d}{1-a} \\
- \frac{(1-(1-\lambda)(1-a)) d}{(1-\lambda)(1-a)} \left[ (1-a)(1-\beta(1-\lambda)(1-a)) + a \right] d.
\]

\[
\geq \frac{d}{(1-\lambda)(1-a)^2} \left[ y_n - y_i - \frac{\eta c \theta}{\beta(1-\eta)} + \frac{\alpha(1-\beta(1-\lambda)(1-a)) d}{1-a} - \frac{(1-(1-\lambda)(1-a)) d}{1-a} \right].
\]

\[
= \frac{d}{(1-\lambda)(1-a)^2} \left[ y_n - y_i - \frac{\eta c \theta}{\beta(1-\eta)} + \frac{\alpha(1-\beta(1-\lambda)(1-a)) d}{1-a} - \frac{(1-(1-\lambda)(1-a)) d}{1-a} \right].
\]
\[
\frac{d}{(1-\lambda)(1-\alpha)}\left[y_h - \gamma - \frac{yc}{\beta(1-\eta)} - (1-\omega)\left[q\lambda - (1-\lambda) + \lambda\right]d\right],
\]

\[
= \frac{d}{(1-\lambda)(1-\alpha)}\left[y_h - \gamma - \frac{yc}{\beta(1-\eta)} - [q\lambda - (1-\lambda) + \lambda]d\right].
\]

This has a positive sign if the following inequality holds:

\[y_h - \gamma - \frac{yc}{\beta(1-\eta)} - [q\lambda - (1-\lambda) + \lambda]d > 0.\] (F-5)

If (F-5) is satisfied, a sign of (F-4) is also positive. Finally we obtain \(\partial\pi_r/\partial\alpha > 0\). The proof is complete.

References


Ravenna, F., and Walsh, C. (2012) “Screening and labor market flows in a model with heterogeneous 
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