Reduction of Work hours, Unpaid Overtime Work and
Time-Varying Unemployment Compensations*

Makoto Masui†

1. Introduction

This paper examines the impact of a reduction in the standard work hours on job creation (we refer to it as "opening a vacancy") under the equilibrium search model with time-varying unemployment compensations that was developed by Albrecht and Vroman (2005). They considered a situation where the unemployment compensation decreases from a high level to a low level as time goes on. Using this model, they obtained some interesting results concerning the job search behavior of unemployed workers. We focus on this research because their model generates wage dispersion in equilibrium without on-the-job search.† Since wage differentials are not negligible for policy makers in industrialized countries, they must be taken into account when planning economic policy. We will examine the effect of a working time reduction that reduces the standard work hours not only on the number of jobs, but also on the wage differentials in equilibrium.

It has generally been expected that a reduction in work hours (work-sharing) expands employment, and in fact, the standard work hours have gradually decreased in several European countries, the U.S. and Japan during the past 30 years.‡ While a great deal of research addresses this subject, whether such a reduction policy actually increases employment is still indeterminate from both theoretical and empirical viewpoints. We explicitly introduce a search friction in the labor market to the model and investigate the impact of the policy on job creation and unemployment.§ While the concept of "job creation" has been

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*I am very grateful to Soka University for its financial support.
†E-mail: mmasui@soka.ac.jp; Tel.: +81—(0)426—91—8914
1) As pointed out in Mincer and Higuchi (1988), Japan has a lower turnover rate than the U.S. In addition, in Barron and Kreps (1999), most graduates do not voluntarily change their initial employers, and the lack of the mobility of labor in Japan is an important factor in heavy investments in firm-specific human capital.
2) The reductions in these countries are precisely described in Contensou and Vranceanu (2000).
3) Mortensen and Pissarides (1994) used the equilibrium search model to examine how employers make decisions to create or destroy a job in response to the level of idiosyncratic productivity shocks. Pissarides (2000) refers to this model in detail. Empirical facts about job creation and destruction in Japan are discussed in Genda (2004).
regarded as extremely important since the 1990s, only a few studies have examined the impact of reducing work hours on job creation. Since it is actually difficult for firms to adjust their employment levels without any pecuniary and non-pecuniary costs, a model that contains these adjustment costs is necessary in order to judge the effectiveness of this reduction policy. This is one of reasons why we use an equilibrium search model to analyze how a policy of working time reduction affects job creation.

Another important factor in this paper is unpaid overtime work. A small number of the papers that have examined the impact of working time reduction on unemployment under the model also addressed the issue of overtime work, and in particular, unpaid overtime work: for example, Bell and Hart (1999a), Hart (2004) and Mizunoya (2005). Mizunoya (2005) is almost unique amongst the literature in that it reports the results of a cross-country comparison of unpaid overtime between several major industrialized countries (Japan, the U.S., the U.K., Germany, and Canada). According to Mizunoya (2005), both male and female full-time workers in Japan engage in the most unpaid overtime work. In particular, Japanese male workers engage in almost twice as much unpaid overtime work as U.S. workers.°° Although some workers will never experience (paid or unpaid or both) overtime work, introducing it into a model enriches any conclusions that can be made concerning the effectiveness of a policy of working time reduction.

Brunello (1989), Trejo (1991), Hunt and Katz (1998), and Bell and Hart (1999a) (1999b) all empirically studied overtime work and examined how regulations for work hours affect employment. Brunello (1989) used data from the Japanese manufacturing sector to demonstrate that a reduction in work hours increases overtime work and reduces employment. Trejo (1991) showed that the effect of a change in the statutory overtime premium is not completely offset by the adjustments of the normal hourly wage rate and suggested that this regulation may expand employment. Hunt and Katz (1998) performed a comparative study of the property of work-sharing performed in the U.S., Germany and France. Using individual data from the German Institute for Economic Research, they showed that a reduction in the 

°° For full-time male workers, the actual work hours and the total overtime hours (paid overtime hours plus unpaid overtime hours) in Japan are the longest amongst the five countries. We have to note, however, that these instances are based on data from the early 1990s.

Mizunoya (2005) also pointed out that in Japan recently, (i) workers who work more than 250 days in a year tend to engage in longer unpaid overtime work (some of these workers work 60 hours per week); on the other hand, (ii) the number of hours worked declines for workers who work fewer than 200 days in a year, and (iii) the number of workers who work more than 250 days in a year decreases. These instances will be related to an increase in the nonregular (or nonstandard) employment. In this model, however, we do not address the difference in the number of overtime hours between workers and between other employment-related categories, such as occupations and industries, for simplicity.
standard work hours leads to a reduction in employment. Bell and Hart (1999a) pointed out that workers in the U.K. experience longer overtime hours than other European countries and offered several reasons why workers engage in unpaid overtime work. Bell and Hart (1999b) investigated how the premium for overtime varies with the passage of time. They showed that the premium in the U.K. is almost invariant, holding steady at about 1.4.

Many theoretical studies that focus on a working-time reduction policy adopt bargaining between employers and a union (or individual workers) as a wage-hours determination mechanism (e.g., Booth and Schiantarelli (1987), Marimon and Zilibotti (2000), FitzRoy et al. (2002) and Rocheteau (2002)). However, according to Cahuc and Zylberberg (2004), union density has gone down in major industrialized countries recently, with the exception of some countries in Northern Europe. Even if labor conditions are determined collectively between the employer and the union, they may not specify the concrete amount of wage payments, but only specify the maximum (minimum) speed of promotion and the annual increase in basic pay (see Aoki (1988)). If so, employers will have a discretionary power of determining wage levels (and other compensations). Furthermore, concerning a bargaining situation between an individual worker and a firm, only workers who have enough skills and abilities to complete tasks by themselves would be able to negotiate work conditions with their employers. Therefore, it is not unreasonable to assume that less skilled workers or workers who are rotated through several related jobs do not have sufficient bargaining power, so employers would take control of determining wages and work hours. For example, like in Japan, an employer may make its employees generalists rather than specialists by rotating workers to several related jobs. Workers hired by this employer will acquire a broad range of skills, but due to this reallocation of employment within the firm, it is difficult to identify the contribution of an individual worker to the firm's business. In this model, we will examine the effectiveness of the policy of reducing work hours under the situation in which firms unilaterally set wages and the number of hours worked. FitzRoy and Hart (1985), Hoel and Vale (1986), Schmidt and Sorensen (1991), and Huang et al. (2004) all addressed this situation, but they did not focus on the search friction and job creation.

As was mentioned earlier, we are concerned with the situation in which a search friction exists in the labor market. Therefore, we construct a model based on the job search model and examine the property of job creation in equilibrium. Marimon and Zilibotti (2000) had motivations similar to ours; they examined the effect of a reduction in work hours on employment by using the equilibrium search model and showed that a mild cut in work hours will generate a small increase in job creation. Since the model developed by Marimon and Zilibotti is based on the standard equilibrium search framework, the wage rate and work
hours are determined by negotiation between a single firm and a single worker. In contrast, we are concerned with the unilateral determination by a firm of wages and work hours. Thus, a different framework is necessary to examine what impact reducing work hours has on job creation under such a unilateral determination mechanism.

For that purpose, we use a wage-posting structure established by Burdett and Mortensen (1998) and Mortensen and Pissarides (1999). Mortensen and Pissarides (1999) developed a model that extends the Burdett–Mortensen framework to introduce elements contained in the equilibrium search model (such as a job matching technology and a flow of jobs). Burdett and Mortensen (1998) and Mortensen and Pissarides (1999) supposed on-the-job search behavior and pointed out that a model generates an equilibrium wage dispersion among homogeneous employers and workers. A main reason for this result is that a high wage payment has two components in the hiring process: the negative effect on profits caused by an increase in per worker cost and the positive effect caused by an increase in applicants and a decrease in turnover. These dual-directional effects provide an infinite number of job offers that make employers indifferent with respect to future profits. Therefore, wage differentials arise in the wage-posting equilibrium. On the other hand, a two-tiered unemployment compensation system is considered in Albrecht and Vroman (2005). They derived a two-point equilibrium wage distribution based on a wage-posting framework without on-the-job search. While most extant literature assumes that the compensation for unemployed workers is constant and permanent, actual unemployment insurance (UI) programs restrict a period of benefits in many industrialized countries. Therefore, we expect that the wage-posting model with time-varying unemployment benefits will provide a different explanation of the effectiveness of a policy of reducing work hours.

5) FitzRoy and Hart (1985) constructed a model with an implicit contract situation and incorporated several forms of taxation intended to levy an unemployment insurance fee. They showed that how a change in the tax rate influences a lay-off level depends on the type of tax. Note, however, that they did not consider a policy of working time reduction. On the other hand, the latter three works constructed each model based on the efficiency wage hypothesis. Hoel and Vale (1986) focused on the relation between job training provided by a firm and employees quitting their jobs. Schmidt and Sorensen (1991) and Huang et al. (2004) called attention to the impact of the length of work hours on productivity of labor. They concluded that this impact crucially depends on the response of work efforts to this policy. In these models, employers make a decision about the number of work hours by considering the behavior of the labor supply side.

6) They also showed that the essential features of the equilibrium are invariable, even if the model contains overtime work.

7) Rocheteau (2002) extended the Marimon-Zilibotti model such that employed workers provide work efforts into production, and these effort levels are the private information of the workers.

8) Countries such as the U.K. and Australia have unemployment assistance systems that do not specify the period of benefits (Belgium has a UI system without time limitation). Oka (2004) made an international comparison of UI systems among several industrialized countries.
The key elements of this paper can be summarized as follows. First, our purpose is to examine the impact of a reduction in the standard work hours on unemployment under a model taking into account unpaid overtime work. Second, our model is based on the equilibrium search model, and we focus on the rate of job creation (which means opening a job vacancy) rather than the number of filled jobs. Third, we suppose that employers unilaterally determine the wage level and work hours. Fourth, the unemployment insurance system in this paper has a time-varying property. Under these settings, we obtain the following results: (i) A reduction in the standard work hours has a positive impact on job creation as long as overtime is moderately compensated. (ii) Under the same situation, such a reduction decreases the ratio of firms offering higher wages. In other words, although this policy expands the opportunities for being employed, it may also increase the number of low wage earners. (iii) The impact of this policy on unemployment is ambiguous. However, even given the promotion of job creation, unemployment does not necessarily decrease. This is due to simultaneous changes in the composition of workers with different reservation wages and the fraction of firms providing a high wage offer. While more job creation reduces unemployment, a rise in the fraction of the high reservation wage and a decline in the fraction of the high-wage job offers increases unemployment because unemployed workers with the high reservation wage only accept high wage offers. Thus, the total effect of reducing the standard work hours is indeterminate.

This paper is organized as follows. In Section 2, we explain the basic structure of the model, including the determination of the optimal work hours and the reservation wages. In Section 3, we demonstrate the existence of an equilibrium in this model. In Section 4, we examine how the policy of reducing work hours affects several economic variables in the equilibrium. Finally, in Section 5, we provide our conclusions.

2. Basic Structures

Matching Technology

There are many identical firms and ex ante identical workers in the economy. The measure of total labor force is denoted by one. Workers are either employed or unemployed. Let $u$ denote the ratio of unemployed workers, and $v$ denote the ratio of vacancy. In this model, only unemployed workers engage in search activity, and vacant firms also seek trading partners. A matching function $m(u, v)$ indicates that the number of matches performed in the search process depends on the numbers of unemployed workers and job vacancies. We suppose that this matching function exhibits constant returns to scale, is increasing with each component, and is concave. The matching function can then be written as $m(u, v)$=.
um(θ), where θ = υ/υ is the labor market tightness. Job seekers accept an offer if it exceeds the reservation wage that makes unemployed workers indifferent between being unemployed and being employed. In this regard, an offer is a random variable with a probability distribution, and therefore the probability that workers meet some offer depends on the shape of this distribution. The arrival rate of jobs for unemployed workers is given by m(u, v)/u = m(θ), and the rate for vacancies is m(u, v)/v = m(θ)/θ. We suppose that m(θ) has the following properties such that

\[ m(0) = 0, m'(θ) > 0, \lim_{θ \to 0} m'(θ) = \infty, \lim_{θ \to 0} \frac{m(θ)}{θ} = \infty, \text{and } \frac{d}{dθ} \left( \frac{m(θ)}{θ} \right) < 0. \]

Workers

This paper is based on the model developed by Albrecht and Vroman (2005). In particular, we focus on the case of risk neutral workers for simplicity. In this model, only unemployed workers engage in searching for jobs, and they receive some benefits from being unemployed. We denote these benefits as b, s and assume that b > s. Note that while b is the high unemployment compensation and all newly unemployed workers start at this level, unemployed workers may receive lower compensation s if they cannot find a job for a long time.

The Bellman equations for unemployed workers with each compensation level, U(b) and U(s), are given by

\[ rU(b) = b + h + \phi m(θ)[N(w_b, l_b) - U(b)] + λ[U(s) - U(b)], \tag{1} \]
\[ rU(s) = s + h + φm(θ)[N(w_s, l_s) - U(s)] + m(θ)(1 - λ)[N(w_s, l_s) - U(s)], \tag{2} \]

where \(N(w_i, l_i)\) for \(i = b, s\) is the value of being employed when workers receive the wage \(w_i\) and work \(l_i\) hours. Let \(φ\) be the fraction of firms providing a high wage and λ be the hazard rate of moving to the lower compensation state. Following Albrecht and Vroman (2005), we suppose that the realization of this event occurs through a Poisson process. h is the value of leisure or of the home production.

Since this model assumes that different levels of unemployment benefits are provided to unemployed workers, we can infer that there are at least two wage offers in the market. Denote the reservation wage for unemployed workers with income \(i (i = b \text{ or } s)\) as \(w_i^r\). Then firms never offer other wage levels (so the actual wage payments \(w_i\) in the above are equal to the reservation wage \(w_i^r\) for \(i = b, s\)). Concretely speaking, firms never offer wages \(w'\) such

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9) From another point of view, we can interpret λ as the hazard rate that an unemployed worker exerts less effort in search activity, and this indolence is penalized such that he or she will subsequently receive a low unemployment benefit.
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that \( w_b < w' < w_b \) or \( w_b < w' \). Even if a firm offers \( w' \in (w_b, w_b) \), only workers with the low reservation wages accept this offer. Thus, the firm could obtain a strictly positive profit by lowering the offer to \( w_b \). Similarly, offering a \( w' \) that is strictly greater than \( w_b \) is never optimal for firms. Job seekers receiving \( b \) thus only accept the offer \( w_b \), and those receiving \( s \) accept both \( w_b \) and \( w_s \).

The Bellman equations for employed workers, \( N(w_b, l_b) \) and \( N(w_s, l_s) \), are

\[
Y_N(w_b, l_b) = w_b + \beta w_b \left( \frac{l_b - I}{l_b} \right) - l_b + \delta [U(b) - N(w_b, l_b)],
\]

\[
Y_N(w_s, l_s) = w_s + \beta w_s \left( \frac{l_s - I}{l_s} \right) - l_s + \delta [U(b) - N(w_s, l_s)],
\]

where \( I \) is the standard work hours and \( l_i \) is the actual (or total) work hours set by a firm.\(^{10}\)

We suppose that \( l_i > I \) for \( i = b, s \); that is, all employers are willing to make employees work strictly longer than the standard working time. This means that a worker is assumed to comply with the regulation of work hours determined by a firm, and he (or she) does not freely choose the number of hours worked. In other words, our model focuses on the situation in which workers are assigned so many tasks that they cannot accomplish the tasks within the standard work hours and work overtime with or without additional payment. In this regard, since working longer hours may raise the likelihood of promotion or of obtaining future wage growth, workers will voluntarily engage in paid or unpaid overtime work to be better off (see Ladners et al. (1996), Anger (2005) and Fannenberg (2005)). However, we leave this topic to a future investigation.

We also suppose that \( I \) is sufficiently high. Denote the flow utility of workers with \( (w_i, l_i) \) as \( x(w_i, l_i) = w_i + [\beta w_i (l_i - I)/I - l_i^2] \) for \( i = b, s \). Note that \( \beta w_i / I \) is the hourly wage rate for overtime hours, \( l_i - I \), and employed workers suffer disutility of labor that is equivalent to \( l_i^2 \).

As in Masui (2009), we suppose that employed workers experience unpaid overtime work and that the size of the parameter \( \beta \) is less than one.\(^{11}\) Furthermore, note that \( w_i \) is not hourly wages, but normal wages for the standard work hours, and that workers who become unemployed obtain the value \( U(b) \) regardless of the income that they received while em-

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\(^{10}\) Strictly speaking, the standard work hours differ from the statutory work hours in Japan and have different values among firms. But, according to Brunello (1989), the normal work hours—that is, the moving average of the standard work hours—can be used as an alternative (time-invariant) indicator.

\(^{11}\) To put it briefly, in a labor market where unpaid overtime work is broadly allowed, employers would not fully compensate the overtime work (with statutory overtime premium). In such a situation, it is plausible to suppose that the premium for compensation of overtime work should be less than one. This supports the assumption that \( \beta < 1 \). Mizunoya (2005) indicated that unpaid overtime is a prevailing phenomenon in many developed countries, including Japan, the U.S., the U.K., Germany and Canada.
ployed. We do not regard wages as an hourly rate because, as Hart (2004) explains, white
collar workers in particular tend to engage in unpaid overtime work. In general, it is more
difficult to capture the hourly performance of these workers than that of blue collar workers.
Therefore, firms are compelled to provide wage payments that are independent of the
number of statutory hours worked.

The reservation property requires that \( N(wb, lb) = U(b) \) and \( N(ws, ls) = U(s) \). This is
because the reservation wage is defined as the wage level that makes workers indifferent
between being employed and unemployed. Since we consider the two types of unemployed
workers, there is a reservation wage level corresponding to each unemployment benefit. It
follows from (3) that \( N(wb, lb) = U(b) = x(wb, lb)/r \). By (1) with \( N(ws, lb) = U(b) \), we have

\[
\left( \frac{r + \delta + \lambda}{\lambda} \right) x(wb, lb) - \frac{(r + \delta)(b + h)}{\lambda} = x(ws, ls).
\]

This is a key expression that describes the relationship between \( \bar{w} \) and \( w_s \). We next redefine
equations (1) and (2) as

\[
\begin{align*}
    rU(b) &= b + h + \lambda[U(s) - U(b)], \\
    rU(s) &= s + h + \phi m(\theta)[U(b) - U(s)].
\end{align*}
\]

Computing (2') - \( rU(b) \) yields

\[
U(s) - U(b) = \frac{s + h - x(wb, lb)}{r + \phi m(\theta)}.
\]

Substituting this into (1') and using \( U(b) = x(wb, lb)/r \) gives

\[
x(wb, lb) = \frac{(b + h)[r + \phi m(\theta)] + \lambda(s + h)}{r + \lambda + \phi m(\theta)}.
\]

Similarly, combining (5) and (6) gives

\[
x(ws, ls) = \frac{\phi m(\theta) - \delta(b + h) + (r + \delta + \lambda)(s + h)}{r + \lambda + \phi m(\theta)}.
\]

Expressions (6) and (7) indicate that the flow utility that workers will receive is a weighted
sum of \( b + h \) and \( s + h \), and more weight is imposed on \( b + h \) in (6). This means that workers
obtain greater utility from receiving \( w_b, l_b \) than from receiving \( w_s, l_s \). Later, we will show
that \( w_b \) is greater than \( w_s \) when \( x(\cdot) \) is monotonically increasing with respect to \( w \). For that
purpose, we need to express the optimal work hours for an employer as a function of \( w_i (i = b, s) \).

Firms

Firms are either vacant or filled. Vacant firms post wages in order to fill vacancies. The
values of having vacancies offering \( w_b \) and \( w_s \), \( \Pi(w_b) \) and \( \Pi(w_s) \), respectively, are given by
(J(w_b, l_b) and J(w_s, l_s), defined below)

\[ r \Pi(w_b) = -c + \frac{m(0)}{\theta} [J(w_b, l_b) - \Pi(w_b)], \]
\[ r \Pi(w_s) = -c + \frac{(1-\gamma)m(0)}{\theta} [J(w_s, l_s) - \Pi(w_s)], \]

where \( \gamma \) denotes the fraction of unemployed workers with the reservation wage \( w_b \). This is an endogenous variable. \( c \) is the cost of maintaining a job. Furthermore, \( m(0)/\theta \) is the arrival rate of job seekers for recruiting firms. Note that firms that offer \( w_s \) only attract unemployed workers who receive the compensation \( s \). On the other hand, the values of filling jobs paying \( w_b \) and \( w_s \), \( J(w_b, l_b) \) and \( J(w_s, l_s) \), are respectively given by

\[ rJ(w_b, l_b) = (l_b)^a - \left[ w_b + \frac{\beta w_b (l_1 - l)}{l} \right] - c + \delta[\Pi(w_b) - J(w_b, l_b)], \]
\[ rJ(w_s, l_s) = (l_s)^a - \left[ w_s + \frac{\beta w_s (l_1 - l)}{l} \right] - c + \delta[\Pi(w_s) - J(w_s, l_s)], \]

The production technology is assumed to be exponential with respect to the total number of hours worked. For simplicity, we suppose that the parameter \( a \), which characterizes the production level of firms, satisfies \( 0 < \alpha < 1 \).  

In the decision-making process, firms determine the optimal work hours that maximize their expected profits given wage levels. Differentiating \( J(w_i, l_i) \) with respect to \( l_i (i=b, s) \) yields

\[ l_i^*(w_i) = \left( \frac{a}{\beta w_i} \right)^{1/(1-\alpha)} \quad \text{and} \quad l_i^*(w_i) = \left( \frac{a l_1}{\beta w_i} \right)^{1/(1-\alpha)}. \]

Denote \( x(w_i) \) as \( x(w_i) = x(w_i, l_1^*(w_i)) \), and similarly, denote \( N(w_i) \) and \( J(w_i) \) as \( N(w_i, l_1^*(w_i)) \) and \( J(w_i, l_1^*(w_i)) = J(w_i) \) for \( i = b, s \).

In this study, the utility of an employed worker does not necessarily increase as his wage rises since it indirectly affects the utility through the work hours given by (12). Although this may provide an interesting implication from the view of compensating differential studies (this is based on the idea that workers will evaluate the value of a job by considering both monetary wage and other non-monetary working conditions, such as fringe benefits), for simplicity we restrict our focus to the case where \( N(w_i) \) is increasing in \( w_i \) for \( i = b, s \). This property holds if the flow utility of employed workers \( x(w_i) \) is increasing in \( w_i (i=b, s) \). Accordingly, since for each \( i \), the derivative of \( x(w_i) \) with respect to \( w_i \), \( x'(w_i) \), is given by

12) Hart (1987) and Contensou and Vranceanu (2000) assumed that the marginal contribution of work hours to production is initially increasing and gradually decreases as time passes.
the sufficient condition for $x'(w_i)$ to be strictly positive is

$$\left(\frac{aT}{\beta w_i}\right)^{(2-a)/(1-a)} - \frac{a^2}{2} > 0,$$

for each $i = b, s$. We suppose that $T$ is sufficiently high such that (14) is satisfied for any $\beta$. It follows from (6), (7) and $b > s$ that $w_b$ is greater than $w_s$. From another point of view, a worker with a higher compensation $b$ has a higher reservation wage. This fact means that we only concentrate on the condition (14) evaluated at $w_b$.

### 3. Equilibrium

To describe a steady-state equilibrium, we must specify conditions for endogenous variables $\gamma$, $u$, $\theta$ and $\phi$. The condition under which the flows into and out of the low-benefit state for unemployed workers are equal gives the steady state ratio, $\gamma$, of unemployed workers with the reservation wage $w_b$:

$$\lambda\gamma u = (1-\gamma)m(\theta)u \Rightarrow \gamma = \frac{m(\theta)}{\lambda + m(\theta)}.$$  

Similarly, the equation for the high benefit state provides the total unemployment rate, $u$, in the steady state:

$$\delta(1-u) = \left[\lambda + \phi m(\theta)\right]u \Rightarrow u = \frac{\delta}{\delta + \gamma[\lambda + \phi m(\theta)]}.$$  

Concerning $\phi$, the equilibrium condition for generating wage dispersion must realize $0 < \phi < 1$. This condition is characterized by the two free entry conditions $\Pi(w_b) = 0$ and $\Pi(w_s) = 0$. In this situation, posting $w_b$ generates the same expected profits as posting $w_s$. So, these posting strategies are indifferent for vacant firms.

The two conditions, $\Pi(w_b) = 0$ and $\Pi(w_s) = 0$, are described by

$$\left(\frac{r + \delta}{m(\theta)}\right)\hat{c}\theta = (t^*)^\theta - \left[w_b + \frac{\beta w_b}{T}(t^* - T)\right]$$

for firms that post $w_b$, and

$$\left(\frac{r + \delta}{\lambda m(\theta)}\right)c\theta = (t^*)^\theta - \left[w_s + \frac{\beta w_s}{T}(t^* - T)\right]$$

for firms that post $w_s$. Expressions (17) and (18) are so-called job creation conditions that are transformations of free entry conditions (for example, see Pissarides (2000)). These conditions determine the equilibrium value of $\theta$. Their left hand sides represent the expected costs of having a vacancy, and their right hand sides are flow profits of having a filled job. Albrecht
and Vroman (2005) specified the condition not only for the dispersion equilibrium, but also for a single-wage equilibrium. Since one of the main concerns in this paper, however, is to investigate the response of the wage differentials to a reduction in work hours, we concentrate on the case where $0 < \phi < 1$. We can specify the condition for a dispersion equilibrium more clearly than the Albrecht-Vroman model because the production level is endogenously determined by the level of work hours. To address the existence and uniqueness of the labor market tightness, we obtain the following proposition.

**Proposition 1**

There is a unique market tightness $\theta$ that satisfies the conditions (17) and (18) if $\theta$ is sufficiently high. Specifically, the sufficient condition for the existence of a unique $\theta$ is given by

$$m(\theta) - (r + \delta)[1 - \frac{\partial m(\theta)}{m(\theta)}] > 0.$$  

(19)

**Proof**

Since $w_b$ in (18) depends on $w_b$ through (5), we first examine the impact of $\theta$ on $w_b$. It follows from (17) that

$$\frac{dw_b}{d\theta} = -c(r + \delta)[m(\theta) - \frac{\partial m(\theta)}{m(\theta)}]\left[\frac{\alpha}{w_b} \left(\frac{\alpha I}{\beta w_b}\right)^{\alpha(1-\alpha)} + (1 - \beta)\right],$$

(20)

where $m(\theta) - \theta m'(\theta) > 0$ due to the linear homogeneity of the matching function. This means that $dw_b/d\theta < 0$. Furthermore, from (5) and (20) it follows that the relationship between $w_b$ and $w_s$ can be described as

$$\frac{dx(w_s)}{d\theta} = x'(w_s)\frac{dw_s}{d\theta} = \left(\frac{r + \delta + \lambda}{\lambda}\right)\frac{dx(w_s)}{d\theta} = \left(\frac{r + \delta + \lambda}{\lambda}\right)x'(w_s)dw_s,$$

and

$$\frac{dw_s}{d\theta} = -\left(\frac{r + \delta + \lambda}{\lambda}\right)x'(w_s)\left[c(r + \delta)[m(\theta) - \frac{\partial m(\theta)}{m(\theta)}]\right]$$

$$\left[\frac{\alpha}{w_b} \left(\frac{\alpha I}{\beta w_s}\right)^{\alpha(1-\alpha)} + (1 - \beta)\right] < 0,$$

(21)

where $x'(w_i)$ is given by (13) for each $i$.

Given that $w_s$ depends on $\theta$ through $w_b$ in (5), each side of (18) is increasing in $\theta$.

$$\text{RHS} \Rightarrow -\left[\frac{\alpha}{w_b} \left(\frac{\alpha I}{\beta w_s}\right)^{\alpha(1-\alpha)} + (1 - \beta)\right]\frac{dw_s}{d\theta} > 0,$$

(13) Note that we see (18) as a single equation for the labor market tightness in the long-run labor market equilibrium.
Therefore, there may exist several \( \theta \) that satisfy the job creation condition (18). We will, subsequently, find a condition of \( \theta \) for a unique solution to (18). Taking the limit as \( \theta \to 0 \), the LHS of (18) goes to zero. This fact results from L'Hopital's rule and the property \( m'(0) \to \infty \) as \( \theta \to 0 \). On the other hand, firms choose their desired levels of work hours to maximize profit for a given \( \theta \). We assume that the firms that pay \( w_s \) obtain strictly positive profits when \( \theta \) goes to zero (small \( \theta \) is favorable because the market is less competitive for firms).

This condition means that the RHS of (18) is strictly positive at \( \theta = 0 \).

Thus, if we can show that the slope of the LHS of (18) is greater than that of the RHS of (18) for every \( \theta \), there exists a unique \( \theta \) that satisfies (18). We will now find a condition for the existence of this unique \( \theta \). Differentiating both sides of (18) with respect to \( \theta \) and subtracting the slope of the RHS from that of the LHS (by using (20) and (21)) yields

\[
\frac{c_r - \delta}{\lambda m'(0)} \left[ m'(0) + \lambda [m(0) - \theta m'(0)] - (r + \delta + \lambda) [m(0) - \theta m'(0)] \right] \geq 0.
\]

where \( Y(w_s) \) and \( Y(w_b) \) are given by

\[
Y(w_s) = \frac{\alpha}{w_s} \left( \frac{a}{\beta w_s} \right) e^{\alpha (1-\beta)} + (1-\beta) \quad \text{and} \quad Y(w_b) = -\frac{\alpha}{w_b} \left( \frac{\alpha}{\beta w_b} \right)^{\alpha (1-\beta)} + (1-\beta).
\]

Regarding the sign of (22) it suffices to specify the condition for realizing the following inequality:

\[
\frac{c_r - \delta}{\lambda m'(0)} \left[ m'(0) + \lambda [m(0) - \theta m'(0)] - (r + \delta + \lambda) [m(0) - \theta m'(0)] \right] \geq 0.
\]

This is because we can show by some calculations that \( Y(w_s) x'(w_s) / Y(w_b) x'(w_b) \) is less than one. The LHS of (18) (denoted as \( S(\theta) \)) and the RHS of (18) (denoted as \( T(\theta) \)), as functions of \( \theta \), then have a unique point of intersection. This argument is depicted in Figure 1. We notice that if the condition

\[
0 \leq m^2(0) - (r + \delta)[m(0) - \theta m'(0)],
\]

\[
= m(0) \left[ m(0) - (r + \delta) \left[ 1 - \frac{\theta m'(0)}{m(0)} \right] \right],
\]

is satisfied, there exists a unique market tightness that satisfies the job creation conditions (17) and (18), assuming the initial condition at \( \theta = 0 \).

Since \( 1 - \theta m'(0) / m(0) \in (0, 1) \), this inequality also holds by assuming \( m(0) > r + \delta \), which is a condition in Albrecht and Vroman (2005) for a unique dispersion equilibrium. This implies that our condition for a unique equilibrium is weaker than the condition in the Albrecht -Vroman model.
To completely describe a unique equilibrium in this model, it is necessary to derive the expression for $\phi$. Once the market tightness satisfying the job creation condition is determined, it follows from (6) that the equilibrium fraction of high wage offers, $\phi$, is also specified for $\theta$ specified in the above. It is given by

$$\phi = \frac{r(b + h) + \lambda(s + h) - (r + \lambda)\pi(w_b)}{m(\theta)[x(w_b) - (b + h)]},$$

(25)

This is the last condition for characterizing a unique market equilibrium. The equilibrium in this model is characterized by (5), (6), (12), (15), (16) and (25). By the job creation conditions (17) and (18), the market tightness $\theta$ and the wage $w_b$ are first determined. This $\theta$ then pins down the fraction of workers, $\gamma$, who obtained a high unemployment benefit. Once $\theta$ and $w_b$ are settled, the optimal work hours $l^*_i$ and the fraction of firms, $\phi$, that offered a high wage offer are determined ($w_b$ and $l^*_i$ are also determined). Finally, the unemployment rate $u$ is settled.

4. The Effect of Reducing the Standard Work Hours

We next examine the effect of reducing the standard work hours, $\bar{I}$, on $\theta, \phi$ and $u$. First, we obtain the following lemma concerning the effect of a change in $\beta$ on wages $w_i$ and $\beta w_i / \bar{I}(i = b, s)$.
Lemma 1

For a fixed $\theta$, an increase in $\beta$ decreases wages $w_i (i = b, s)$. Furthermore, the hourly wage rate for overtime work $(\beta w_i T)$ increases as $\beta$ increases.

Proof

It follows from (17) and (18) that for a fixed $\theta$, we have

$$\frac{\partial w_i}{\partial \beta} = \left[ -\frac{a}{\beta} \left( \frac{aT}{\beta w_i} \right)^{\alpha/(1-\alpha)} + w_i \right] / \left[ \frac{a}{w_i} \left( \frac{aT}{\beta w_i} \right)^{\alpha/(1-\alpha)} + (1-\beta) \right].$$ \hspace{1cm} (26)

The numerator in (26) is negative because we have assumed that $\mu^* > \bar{T}$ for each $i$ (that is, all employed workers work longer than the mandatory standard work hours). Therefore, the overall sign of (26) is also negative. This means that an increase in $\beta$ results in higher labor costs, and firms react to this change by reducing the normal wage $w_i$. On the other hand, since $\partial (\beta w_i) / \partial \beta = w_i + \beta \partial w_i / \partial \beta$, we have from (26) that

$$\frac{\partial (\beta w_i)}{\partial \beta} = w_i \left[ \frac{a}{w_i} \left( \frac{aT}{\beta w_i} \right)^{\alpha/(1-\alpha)} + (1-\beta) \right] > 0.$$ \hspace{1cm} (27)

Thus, the hourly wage rate for overtime work goes up as $\beta$ increases.

By using this lemma, we arrive at the following proposition.

Proposition 2

(i) If the parameter $\beta$ is at a moderate level such that the following condition is satisfied:

$$\left( \frac{aT}{\beta w_b} \right)^{(2-\alpha)/(1-\alpha)} \leq \frac{a}{2} \leq \left( \frac{aT}{\beta w_s} \right)^{(2-\alpha)/(1-\alpha)},$$

then more jobs will be created as the standard work hours are reduced. That is, $\partial \theta / \partial \bar{T} < 0$.

(ii) If $\beta$ is so high (or low) that the condition

$$\left( \frac{aT}{\beta w_b} \right)^{(2-\alpha)/(1-\alpha)} < \frac{a}{2} \text{ or } \left( \frac{aT}{\beta w_s} \right)^{(2-\alpha)/(1-\alpha)} > \frac{a}{2},$$

is satisfied, then the effect that a reduction in $\bar{T}$ has on the creation of jobs is ambiguous. Furthermore, the interval in which a reduction in work hours has a desirable effect, $[\beta_1, \beta_2]$, shrinks as $\lambda$ becomes large, where $\beta_1$ satisfies $(aT/\beta_1 w_b)^{(2-\alpha)/(1-\alpha)} = a/2$ and $\beta_2$ satisfies $(aT/\beta_2 w_s)^{(2-\alpha)/(1-\alpha)} = a/2$.

Proof

From (18), we can derive the reaction of an equilibrium market tightness to a change in the standard work hours:
\[
\frac{\partial \theta}{\partial \bar{t}} = \left[ \frac{a^2}{\beta (\bar{\theta} \bar{w}_b)^{(2\sigma-1)/(1-\sigma)}} \frac{w_s - \bar{I} \partial w_b}{w_s} - (1-\beta) \frac{\partial w_b}{\partial \bar{t}} \right] \\
\left[ \frac{d \text{LHS of (18)}}{d \theta} - \frac{d \text{RHS of (18)}}{d \theta} \right].
\]  

(29)

We have previously seen the relationship between \( w_s \) and \( w_b \) in (5). This results in

\[
\frac{\partial w_b}{\partial \bar{t}} = \frac{1}{x'(w_b)} \left[ \left( \frac{r+\delta+\lambda}{\lambda} \right) \frac{\partial x(w_b)}{\partial \bar{t}} - \frac{\partial x(w_b)}{\partial w_b} \right]_{w_s \text{fixed}}.
\]

(30)

In short, a change in \( \bar{t} \) has an impact on \( w_s \) through \( w_b \). With respect to \( \partial x(w_b)/\partial \bar{t} \) in (30), it is given from the definition of \( x(w_b) \) by

\[
\frac{\partial x(w_b)}{\partial \bar{t}} = \left. \frac{\partial x(w_b)}{\partial \bar{t}} \right|_{x(w_b) \text{ fixed}} + x'(w_b) \frac{\partial w_b}{\partial \bar{t}}.
\]

(31)

Note that \( \bar{t} \) has both direct and indirect effects on \( x(w_b) \). Furthermore, using (17) we can obtain \( \partial w_b/\partial \bar{t} \) as follows:

\[
\partial w_b \left[ -\frac{a}{\bar{w}_b} \left( \frac{a \bar{t}}{\beta \bar{w}_b} \right)^{\sigma(1-\sigma)} \right. - (1-\beta) \left. \right] + \partial \bar{t} \left[ \frac{a}{\bar{w}_b} \left( \frac{a \bar{t}}{\beta \bar{w}_b} \right)^{\sigma(1-\sigma)} \right] = 0,
\]

\[
= \frac{\partial w_b}{\partial \bar{t}} = \frac{a}{\bar{t} \left( \frac{a \bar{t}}{\beta \bar{w}_b} \right)^{\sigma(1-\sigma)}} \left[ \frac{a}{\bar{w}_b} \left( \frac{a \bar{t}}{\beta \bar{w}_b} \right)^{\sigma(1-\sigma)} \right. + (1-\beta) \left. \right] > 0.
\]

(32)

The definition of \( x(w_b) \) then provides us the exact form of (31) by using (32):

\[
\frac{\partial x(w_b)}{\partial \bar{t}} = -\frac{2(1-\beta)}{\bar{t}(1-\alpha)} \left( \frac{a \bar{t}}{\beta \bar{w}_b} \right)^{\sigma(1-\sigma)} \left[ \left( \frac{a \bar{t}}{\beta \bar{w}_b} \right)^{(2-\sigma)/(1-\sigma)} \right. - \frac{a}{2} \left. \right] \\
\left. \right/ \left[ \frac{a}{\bar{w}_b} \left( \frac{a \bar{t}}{\beta \bar{w}_b} \right)^{\sigma(1-\sigma)} \right. + (1-\beta) \left. \right].
\]

(33)

The sign of (33) depends on the size of \( (a \bar{t}/\beta \bar{w}_b)^{(2-\sigma)/(1-\sigma)} \), which appears in the numerator of (33).

On the other hand, we note that arranging the numerator of the RHS in (29) results in

\[
\frac{a}{\bar{t} \left( \frac{a \bar{t}}{\beta \bar{w}_b} \right)^{\sigma(1-\sigma)}} - \frac{\partial w_b}{\partial \bar{t}} \left[ \frac{a}{\bar{w}_b} \left( \frac{a \bar{t}}{\beta \bar{w}_b} \right)^{\sigma(1-\sigma)} \right. + (1-\beta) \left. \right].
\]

(34)

Since \( \partial x(w_b)/\partial \bar{t} \)|\( w_s \text{ fixed} \) is given by

\[
\frac{\partial x(w_b)}{\partial \bar{t}} \left|_{w_s \text{ fixed}} \right. = \frac{2}{\bar{t}(1-\alpha)} \left( \frac{a \bar{t}}{\beta \bar{w}_b} \right)^{\sigma(1-\sigma)} \left[ \frac{a^2}{2} - \left( \frac{a \bar{t}}{\beta \bar{w}_b} \right)^{(2-\sigma)/(1-\sigma)} \right],
\]

(35)

it follows from the expressions (30)–(35) that we can obtain the expression of (34)\(^{14}\)

\(^{14}\) Notice that we first exclude the term \( \frac{r+\delta+\lambda}{\lambda} \frac{\partial x(w_b)}{\partial \bar{t}} \) from the calculation.
The complete expression of (34) then becomes

$$
\frac{\alpha^2}{\beta w_s} \left( \frac{\alpha I}{\beta w_s} \right)^{(2-a)/(1-a)} + \frac{1}{x'(w_s)} \left[ \frac{a}{\beta w_s} \left( \frac{a I}{\beta w_s} \right)^{(2-a)/(1-a)} + (1-\beta) \right] \frac{\partial x(w_s)}{\partial I}_{w_s, fixed},
$$

$$
= \frac{1}{x'(w_s)} \left( \frac{2(1-\beta)}{(1-a)} \right) \left( \frac{a I}{\beta w_s} \right)^{(2-a)/(1-a)} \left[ \frac{a}{2} - \left( \frac{a I}{\beta w_s} \right)^{(2-a)/(1-a)} \right].
$$

It follows from Lemma 1 that $X_i(\beta) = (\alpha I/\beta w_s)^{(2-a)/(1-a)} (i = b, s)$ is decreasing in $\beta$. Thus, the functions $X_b(w_b)$ and $X_s(w_s)$ are downward-sloping in $(\beta, X)$ space (see Figure 2). Furthermore, the fact that $w_s < w_b$, which is confirmed at the end of Section 2, indicates that $X_b(\beta) < X_s(\beta)$ for every $\beta$. By definition of $\beta_b$ and $\beta_s$, if $\beta_1 < \beta \leq \beta_2$, then $X_s(\beta) < a/2 \leq X_b(\beta)$ and $\partial \theta \partial I \leq 0$ (a sign of the denominator in (29) is positive). On the other hand, if $\beta_b < \beta < \beta_1$, then $X_s(\beta) > a/2$ or $X_b(\beta) < a/2$ and the sign of $\partial \theta \partial I$ is indeterminate. Additionally, it follows from (5) and (17) that the moving rate $\lambda$ raises $w_s$ and has no impact on $w_b$ because $\theta$ is fixed in this calculation ($\lambda$ does not appear in (17)). Therefore, $\beta_b$ declines and $\beta_s$ is unaffected as $\lambda$ rises. In other words, (36) is less likely to be negative, and a higher value of $\beta$ makes the policy of reducing the standard work hours less attractive because the area of $\beta$ that realizes...
$\partial \theta / \partial \ell < 0$ becomes small. The proof is completed. ■

Proposition 2 indicates that a reduction in work hours will have a favorable effect on job creation when $\beta$ is at a moderate level. The interpretation of this result can be described as follows. First, the reduction in $\ell$ indirectly affects the flow profit of a firm appearing in (18) through $w_s$. This effect takes place through two paths: one is through $x(w_s)$ and the other is through $x(w_b)$. It follows from (30) and (35) that the effect of reducing $\ell$ through $x(w_s)|w_s$: fixed is positive, and this in turn makes $w_s$ decrease. Concerning the effect on $w_e$, it follows from (33) that $x(w_b)$ falls with the reduction in $\ell$ when $\beta$ is so high that $\beta$ is greater than $\beta_1$.

In this instance, the income, $\beta w_b$, that workers obtain from working overtime increases as $\beta$ rises by Lemma 1. Then, $x(w_s)$ declines because an opportunity cost of not working overtime for employed workers becomes large by a reduction in $\ell$. Therefore, while reducing $\ell$ decreases disutility from labor, this effect is negated by the drop in the normal wage $w_b$.

The second term of the RHS in (31) is thus greater than the first term, and as a result, $x(w_b)$ declines due to a reduction in $\ell$. Accordingly, the reservation wage $w_s$ will fall by reducing the standard work hours, and this reduction in $w_s$ increases job creation because it follows from (18) that profits of firms represented by the RHS of (18) rise with the reduction in $w_s$.

Second, a reduction in $\ell$ has a direct impact on the flow profit of the firm as shown in (18) with fixed $w_s$. This impact is negative because for a fixed $w_s$, a decline in $\ell$ means that the firm must make its employees work fewer hours without a decrease in compensation, which reduces the revenue. Thus, creating jobs is not favorable for potential firms in this aspect.

In the proof of Proposition 2, we finally compare the former indirect effect with the latter direct effect in order to determine whether or not the reduction in the standard work hours generates more jobs. Since the marginal product of labor $(\sigma l^*)^{\sigma -1}$ becomes small for fixed $w_s$ when $\beta$ is small, a reduction in the flow profit described by the direct effect of $\ell$ will be sufficiently small. In addition,

15) This means that a country with an unemployment compensation system with a low moving rate will achieve high job creation by reducing the standard work hours. As has been stated, countries such as Australia, Belgium and the U.K. have unemployment insurance or assistance systems that continue paying benefits for indefinite periods.

16) Total number of hours worked $l^*$ declines when $\ell$ is reduced. Therefore, reducing the standard work hours decreases the actual number of hours worked for employed workers receiving $w_s$. This can be shown as follows. It follows from (32) that concerning the hourly wage rate $w_s/l^*$, we have

$$
\frac{\partial w_s}{\partial l^*} = \frac{1}{l^*} \left( \frac{\partial w_s}{\partial \ell} - \frac{\partial w_s}{\partial l} \right),
$$

$$
= -(1 - \beta) l^* \left( \frac{\sigma l^*}{w_s} \left( \frac{\sigma l^*}{w_s} \right)^{\sigma(\sigma -1)} + (1 - \beta) \right) < 0.
$$

Since $l^*$ is a decreasing function with respect to $w_s/l^*$, the reduction in $\ell$ results in lower $l^*$. That is, employed workers with higher wages will work shorter hours if the standard work hours are reduced.
is larger as $\beta$ becomes smaller, and then, reducing $\bar{t}$ results in cutting wage costs. This implies that the latter effect is greater than the former effect if $\beta$ is less than $\beta_2$.

In the end, it can be seen that reducing the standard work hours unambiguously creates more jobs only when overtime work is compensated moderately. In Masui (2009), on the other hand, a reduction in the standard work hours increases job creation if overtime work is insufficiently compensated (that is, if $\beta$ in this model is sufficiently small).

An increase in $\lambda$ expands the proportion of unemployed workers with the benefit $s$. This is favorable for firms posting $w_s$ because they can make matches after a shorter waiting period. Then, the expected costs of having a vacancy will fall, and potential firms will be willing to have a vacancy by posting $w_s$. The free entry condition then results in raising $w_s$ to decrease profits by having this type of job. As a result, this is achieved by an increase in $w_s$, and then $X_s(\beta)$ (therefore $\beta_2$) goes down. For a given $\theta$, $w_b$ is not affected by a change in $\lambda$.

The next proposition states that a reduction in $\bar{t}$ decreases $\phi$ when $\partial \theta / \partial \bar{t}$ is negative.

**Proposition 3**

*When the standard work hours are reduced, then the fraction of firms that provide a high wage offer, $\phi$, declines. In short, the sign of $\partial \phi / \partial \bar{t}$ is positive if $\partial \theta / \partial \bar{t}$ is negative at $\beta \in [\beta_1, \beta_2]$.**

**Proof**

Expression (25) enables us to compute $\partial \phi / \partial \bar{t}$. Differentiating $\phi$ with respect to $\bar{t}$ results in

$$\frac{\partial \phi}{\partial \bar{t}} = \frac{\partial \phi}{\partial \theta} \frac{\partial \theta}{\partial \bar{t}} + \frac{\partial \phi}{\partial x(w_b)} \frac{\partial x(w_b)}{\partial \bar{t}} \bigg| _{\theta \text{fixed}}. \quad (37)$$

Since we already know the form of $\partial x(w_b) / \partial \bar{t} | _{\theta \text{fixed}}$ by (33), it is enough to compute $\partial \phi / \partial \theta$ and $\partial \phi / \partial x(w_b)$ to specify the sign of (37). First, it follows from (25) that the derivatives of $\phi$ with respect to $\theta$ are given by

$$\frac{\partial \phi}{\partial \theta} = \frac{-m(\theta)(x(w_b)-(b+h))\partial x(w_b)/\partial \theta}{m^2(\theta)[x(w_b)-(b+h)]^2} \left[ x(b+h)+\lambda(s+h)-(x+\lambda)x(w_b) \right] \left[ m(\theta)[x(w_b)-(b+h)] + m(\theta)\partial x(w_b)/\partial \theta \right].$$

The numerator of this expression becomes
\[
\lambda m(\theta)(b-s)\frac{\partial x(w_b)}{\partial \theta} - [r(b+h) + \lambda(s+h) - (r+\lambda)x(w_b)]m'(\theta)[x(w_b)-(b+h)]. \tag{38}
\]

(38) has a negative value because (i) \(\partial x(w_b)\partial \theta\) is negative, (ii) \(x'(w_b)>0\), (iii) \(r(b+h) + \lambda(s+h) - (r+\lambda)x(w_b)<0\) and (iv) \(x(w_b)-(b+h)<0\). The last result derives from the property of \(x(w_b)\) in (6). On the other side of the equation, we have

\[
\frac{\partial \phi}{\partial x(w_b)} = \frac{-(r+\lambda)[x_b(\theta)-(b+h)] - [r(b+h) + \lambda(s+h) - (r+\lambda)x_b(\theta)]}{m(\theta)[x(w_b)-(b+h)]^2} > 0. \tag{39}
\]

It follows from (33), (37), (38) and (39) that if \(\beta_1 \leq \beta \leq \beta_2\), we have

\[
\frac{\partial \theta}{\partial \tilde{I}} < 0, \quad \frac{\partial x(w_b)}{\partial I} \bigg|_{\beta=\beta_0} > 0 \quad \text{and} \quad \frac{\partial \phi}{\partial \tilde{I}} > 0. \]

Proposition 2 indicates that if \(\beta\) takes a moderate value, then the reduction in work hours will create more jobs in the economy. At the same time, however, Proposition 3 indicates that this policy increases the ratio of jobs that provide low wage payments. By Eq. (37), the change in \(\tilde{I}\) has effects on \(\phi\) through \(x(w_b)\) both directly and indirectly. In this regard, from (6), \(x(w_b)\) is given as a convex combination of \(b+h\) and \(s+h\), and \(x(w_b)\) becomes greater as \(\phi\) increases. Since \(w_b\) is the reservation wage, a rise in the utility obtained from being hired must be accompanied by an increase in the utility from being unemployed (since \(N(w_b)=U(b)\) must hold in equilibrium ). This is achieved through the rise in \(\phi\). Since those effects are negative when \(\tilde{I}\) declines, the total effect is also negative.17

The next proposition states the results about the impact of reduction policy on the structure of the unemployment pool and the unemployment rate.

**Proposition 4**

Suppose that the standard work hours are reduced. It then follows from (15) and (16) that (i) the fraction of workers receiving the larger unemployment benefit, \(\gamma\), increases; and (ii) the equilibrium unemployment rate, \(u\), moves ambiguously.

**Proof**

Concerning \(\gamma\) and \(u\), it follows from (15) and (16) that we have

\[
\frac{\partial \gamma}{\partial \tilde{I}} = \frac{\lambda m(\theta)}{[\lambda + m(\theta)]^2} \frac{\partial \theta}{\partial \tilde{I}} < 0, \quad \text{if} \quad \beta \in [\beta_1, \beta_2]. \tag{40}
\]

17 On the firm side, a reduction in \(\tilde{I}\) increases the ratio of overtime hours compared to total number of hours worked. This results in higher labor costs for firms that pay \(w_b\) because of the higher hourly wage rate for overtime work, \(\beta w = \tilde{I}\) when \(\beta\) and \(w\) are high (\(\beta > \beta_1, w_b > w_b\)). These instances make posting \(w_b\) less attractive. Hence, the ratio \(\phi\) declines when \(\tilde{I}\) is reduced.
\[ \frac{\partial u}{\partial \bar{t}} = -\delta \left[ \lambda + \phi m(\theta) \right] \frac{\partial \gamma}{\partial \bar{t}} + \gamma \frac{\partial \phi m(\theta)}{\partial \bar{t}} \right] / \left[ \delta + \gamma \left( \lambda + \phi m(\theta) \right) \right]^2. \] (41)

Since \( \phi \) is given by (25), we can calculate \( \frac{\partial \phi m(\theta)}{\partial \bar{t}} \) as follows:

\[ \frac{\partial \phi m(\theta)}{\partial \bar{t}} = \frac{\partial}{\partial \bar{t}} \left[ \frac{x(b + h) + \lambda(s + h) - (x + \lambda)x(w_b)}{x(w_b) - (b + h)} \right]. \]

\[ = \frac{\lambda(b - s)}{\left[ x(w_b) - (b + h) \right]^2} \frac{\partial x(w_b)}{\partial \bar{t}}. \] (42)

The above expression depends on \( \frac{\partial x(w_b)}{\partial \bar{t}} \), and so, (20) and (33) result in

\[ \frac{\partial x(w_b)}{\partial \bar{t}} = \frac{\partial x(w_b)}{\partial \bar{t}} \bigg|_{\theta_{\text{fixed}}} + \frac{x(w_b) \partial w_b}{\partial \theta} \frac{\partial \theta}{\partial \bar{t}} > 0 \quad \text{if} \quad \theta \in [\beta_1, \beta_2]. \] (43)

This is different from (31) or (33) in that (43) includes the impact through \( \theta \). We can then conclude that if \( \beta \) is contained in the interval \([\beta_1, \beta_2] \),

\[ \frac{\partial x(w_b)}{\partial \bar{t}} > 0, \quad \frac{\partial \phi m(\theta)}{\partial \bar{t}} > 0 \quad \text{and} \quad \frac{\partial u}{\partial \bar{t}} > 0. \] (44)

The proof is completed.

Concerning the first statement in this proposition, we first stated that the proportion of unemployed workers with the reservation wage \( w_b \) rises with the increase in market tightness. From Proposition 3, we know that \( \phi m(\theta) \) is decreasing with respect to \( \theta \). The outflow of those workers from the unemployment pool then decreases, while the outflow of workers with the low reservation wage increases as \( \theta \) goes up (because \( m'(\theta) > 0 \)). Then, unemployed workers with compensation \( b \) tend to remain in the unemployment pool since the fraction of high wage jobs is smaller than before. This implies that the fraction of workers with the higher reservation wage, \( \gamma \), becomes high in the unemployment pool.

We will next provide an explanation of how the reduction policy will affect the unemployment rate. We notice from (41) that the change in the standard work hours has an impact on \( u \) through \( \gamma \) and \( \phi m(\theta) \). Provided that the result \( \partial \theta / \partial \bar{t} < 0 \) is obtained, we have \( \partial \gamma / \partial \bar{t} < 0 \) and \( \partial \phi m(\theta) / \partial \bar{t} > 0 \). Note first that the outflow from the unemployment pool decreases with an increase in \( \gamma \). This outflow is expressed by

\[ \gamma \phi m(\theta) u + (1 - \gamma) [\phi m(\theta) + m(\theta)(1 - \phi)] u = [1 - \gamma (1 - \phi)] m(\theta) u. \]

This is obviously decreasing with respect to \( \gamma \). It follows from the definition of \( \gamma \) that a rise in \( \gamma \) means that the number of unemployed workers who seek only a job with \( w_b \) increases. This reduces the job finding rate of those unemployed workers because (i) they face more competition for getting a well-paid job, and (ii) it follows from Proposition 3 that the proportion of this kind of job \( \phi \) declines as \( \bar{t} \) is reduced (furthermore, \( \phi m(\theta) \) also declines).
So these effects will increase unemployment. On the other hand, since the job finding rate for unemployed workers whose reservation wage is \( w_s \) is \( m(\theta) \), a reduction in \( \bar{I} \) raises this rate and will decrease unemployment because we restrict our analysis to the situation in which \( \partial \theta / \partial \bar{I} < 0 \) holds. Hence, a reduction in \( \bar{I} \) has an ambiguous effect on the unemployment rate.\(^{18}\)

We finally note that the adjustment in \( b \) and \( s \) will improve the effect of the reduction policy. Concretely speaking, it follows from (5) that an increase in \( b \) reduces \( w_s \), and therefore, this makes \( \beta_2 \) higher for a fixed \( \theta \). Since the expression (17) indicates that the increase in \( b \) has no impact on \( w_s \) when \( \theta \) is given, the interval \([\beta_1, \beta_2]\) expands. That is, the equilibrium market tightness is more likely to increase by reducing \( \bar{I} \) (a lower \( w_s \) is attractive for potential employers). On the other hand, a rise in \( s \) decreases \( \partial m(\theta) / \partial \bar{I} \) in (42). As in the Albrecht-Vroman model, since a change in \( s \) does not influence \( \theta \), the impact of \( \bar{I} \) through \( \phi m(\theta) \) on \( u \) becomes small by raising \( s \) and diminishing \( b - s \). Then, the policy of the reduction in the standard work hours may simultaneously realize the expansion in job creation and the decline in the equilibrium unemployment rate.

5. Conclusions

We developed an equilibrium search model with time-varying unemployment compensations. Specifically, our model is based on the wage-posting framework to examine how a reduction in the standard work hours affects some important economic variables, such as the level of job creation (labor market tightness) and the unemployment rate. In this paper, wage levels are unilaterally determined by employers and wage offers are posted before the search process begins. Furthermore, unemployed workers in this model are compensated differently, according to a two-tiered system. Two types of reservation wages endogenously arise, which allows for the generation of a wage dispersion in equilibrium without on-the-job search. In these circumstances, we obtain the following results: (i) a reduction in the standard work hours has a positive impact on job creation as long as overtime is moderately compensated; under the same situation in which job creation is promoted, (ii) the ability to create new jobs decreases as the rate of moving to a lower compensated state rises; (iii) a reduction in the standard work hours decreases the ratio of firms offering higher wages; and (iv) the impact of this policy on the unemployment rate is ambiguous. It is important to note that despite the creation of new jobs, unemployment does not necessarily decrease because the policy also changes the composition of workers in the unemployment pool and the fraction of firms

\(^{18}\) Masui (2009) does not contain the effect on \( u \) through \( \gamma \) and \( \phi \). Therefore, an increase in vacancies affects the unemployment rate only through the matching frequency (i.e., \( m(\theta) \) in this model). As a result, a reduction in the standard work hours unambiguously diminishes the steady state unemployment rate.
providing different wage offers. In summary, while reducing work hours has the potential to encourage job creation, this policy may at the same time increase the ratio of low-income persons. Finally, we must point out that the effectiveness of the policy depends on the properties of the unemployment compensation system, such as the moving rate and the compensation levels.

References


Masui, M. 2009, 'Policy of Working Time Reduction, Unpaid Overtime Work and Unemployment,' Soka


